Specialization in a Knowledge Economy

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Abstract

Using firm-level data from the US Census Longitudinal Business Database (LBD), this paper exhibits novel evidence about a wave of specialization experienced by US firms in the 1980s and 1990s. Specifically: (i) Firms, especially innovating ones, decreased production scope, i.e., the number of industries in which they produce. (ii) Innovation and production separated, with small firms specializing in innovation and large firms in production. Higher patent trading efficiency and stronger patent protection are proposed to explain these phenomena. An endogenous growth model is developed with potential mismatches between innovation and production. Calibrating the model suggests that increased trading efficiency and better patent protection can explain 20% of the observed production scope decrease and 108% of the innovation and production separation. They result in a 0.64 percent point increase in the annual economic growth rate. Empirical analyses provide evidence of causality from pro-patent reforms in the 1980s to the two specialization patterns.
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Keywords: specialization, production scope, R&D, intellectual property rights, patent trade, endogenous growth

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1 Introduction

Profiting from innovation is vital for the survival of innovating firms and, therefore, economic growth. However, it is not easy to monetize innovation using a firm's own production. First, ideas are random and are not always matched with a firm's production.[1] Second, the firm may lack the ability to mass-produce its innovation output.[2] Strategies to solve these problems within the firm include: spanning a large number of industries to raise the opportunity of utilizing new inventions; doing innovation only when the firm can produce and commercialize new inventions.

Surprisingly, this paper finds deviations from the aforementioned strategies among US firms in the 1980s and 1990s using the Longitudinal Business Database (LBD) from the Census Bureau—there is novel evidence of specialization trends.[3] Specifically,

(i) US firms narrowed their production scopes, i.e., the number of industries in which they produce. The scope shrinkage was driven by innovating firms.

(ii) Innovation shifted from large firms (firms with mass production) to small firms. This study then asks: What are the driving forces of the observed specialization, and how do they affect economic growth?

This paper proposes that higher patent trading efficiency and better patent protection contribute to the specialization patterns by allowing innovations to be traded and utilized by other firms. To assess this new hypothesis in explaining the specialization choices of US firms and economic growth, an endogenous growth model is built with potential mismatches between innovation and production and firm heterogeneity in the ability to monetize new inventions through production. Then, the model is calibrated to rich firm-level data from the LBD, R&D data from the Survey of Industrial Research and Development (SIRD), and patent data from the US Patent and Trademark Office (USPTO). The model suggests that increased patent trading efficiency and patent protection can jointly explain 20% of the production scope contraction and 108% of the shift of innovation activities. They lead to a 0.64 percent point increase in the annual economic growth rate.

Here is a complete summary of the hypothesis. Increased patent trading efficiency and patent protection made innovations more commodified and tradable. Trading of innovations on the patent market allowed firms to sell the new inventions that fell outside of their production scope and buy inventions that could be utilized by their production;

[1] Akcigit, Celik and Greenwood (2016) provides evidence that firms may generate new inventions that are far away from the firms' primary line of business. In this case, the inventions have less value to the firms.

[2] For example, RC Cola was a small beverage company that introduced the first cola in a can and the first diet cola. However, it quickly lost the advantage to Coca-Cola and Pepsi. De Havilland, the world's first commercial jet airliner, invented the Comet I jet 2 years before Boeing introduced the 707. However, de Havilland was not able to capitalize its early invention. For more examples, please see Teece (1986)

[3] The LBD covers all US firms with paid employees.

thus, making firms' production scope contribute less to the value of their innovation. This explains why innovating firms sharply decreased production scope in the 1980s and 1990s (Fact i). Small firms often have limited ability to monetize innovation through their own production. Chances of selling innovation output on the patent market benefited them more and incentivized them to increase innovation efforts. Large firms could rely on small firms' innovation by purchasing patents on the market and therefore decreased innovation efforts. This explains why innovation activities shifted to small firms (Fact ii).

Two pieces of evidence provide direct support for the new hypothesis. First, the volumes of patent trading activities ballooned after the early 1980s. According to the Patent Assignment Dataset (PAD) from the USPTO, the citation-weighted share of patents traded within 10 years of issuance increased from 23.2% in 1983 to 37.0% in 2000. This increase shows that innovations have become more tradable. Second, the average matching rate between the technology class of a patent and its inventing firm's industry class declined from 3.8% in 1981 to 2.2% in 2000.[4] It suggests fewer innovations were utilized by the firms that invented them.

The 1980s and 1990s witnessed two major changes related to patent trading—the rise of information technology and a series of pro-patent reforms. Improvement in information technology allowed the USPTO to deploy the first automated search systems for trademarks and patents in the 1980s, which significantly raised search capability and reduced information frictions in trade. The pro-patent reforms include an extension of patentability to genetic engineering and software and the creation of the Court of Appeals for the Federal Circuit (CAFC) that vastly increased the winning opportunity of patent holders in legal disputes by lowering invalidation rates. On the one hand, these reforms incentivized firms to patent their inventions instead of hiding them as secrets, therefore, decreased information frictions in trading innovation. The effect of patent protection on patent trade through information disclosure is discussed in Lamoreaux and Sokoloff (2001) using historical data. On the other hand, those reforms allowed firms with new inventions to extract more value in the trading process since it was less likely that the potential buyers would use legal disputes to get the patent for free.

Other possible explanations are also considered for the observed specialization patterns. First, the US government introduced a R&D tax credit in 1981 as part of the strategies to increase the competency of US firms in the global market. The effective federal subsidy rate increased from 5% before the 1980s to 24% in the 1990s, as documented in Akcigit, Ates and Impullitti (2018). Combined with the booming patent trading market,

[4] The technology class of a patent is based on the 4-digit code of the International Patent Classification (IPC); the firms' industry class is based on the 6-digit NAICS code. This paper builds a concordance between the two using the method in Silverman (2002) and a link between the SIC and NAICS codes. Silverman (2002) bridges the patent technology classes with industries according to the usage of the technology.

the R&D tax credit may have benefited small firms more as their R&D expense to domestic sales ratio grew to be higher than large firms' after 1985. Therefore, the tax credit may have amplified the shift of innovation to small firms.[5] Second, the cost structure of production may have changed over time that directly affected firms' production scope. Recent papers like Hsieh and Rossi-Hansberg (2019) and De Ridder (2019) argue that the rise of information technology increases the fixed cost for firms to enter new industries but decreases the marginal production cost after entry. This may explain the observed shrinkage of production scope.[6] Third, good ideas may be getting harder to find, as argued by Bloom et al. (2020). This may have pushed innovating firms to focus efforts on narrower fields of research and therefore production.

To evaluate the roles of the new hypothesis, as well as the aforementioned possible explanations in the specialization patterns and economic growth, a quantitative model is built with endogenous decisions of production scope and innovation effort. Distinct from existing theories about innovation (e.g., Garcia-Macia, Hsieh and Klenow (2019)) where the benefit from new ideas does not depend on production scope, the model in this paper takes into account potential mismatches between innovation output and production. A key tradeoff that an innovating firm faces when choosing its production scope is that larger scope raises the probability that the firm's innovation output is better matched with its production and, therefore, increases the firm's ability to monetize its inventions; but at the same time, larger scope increases the management cost of the firm. The patent trading market provides another channel for firms to benefit from their innovation besides production but is subject to search frictions. When the matching efficiency increases and the invalidation rate of patents in legal disputes decreases, the relative importance of production versus trading in monetizing innovation changes. The effects are heterogeneous for small and large firms. Small firms have limited production scope and benefit more from selling patents; large firms have broader scope and benefit more from buying patents. The model also entertains other explanations.

The developed model is first calibrated to an initial balanced growth path (1981-1985) using the LBD, the SIRD, and the USPTO patent datasets. Key calibration targets include production scope, the R&D expense-to-domestic sales ratio of large and small firms, the share of patents traded, and the HP-filtered economic growth rate. Then, the model is recalibrated to fit an ending balanced growth path (1996-2000), allowing changes in parameters relevant to the new hypothesis and the three alternative explanations. A decomposition exercise is conducted to explore the contribution of each possible explanation by

[5] A further discussion of the impact of R&D subsidy and taxation policies can be found in Atkeson and Burstein (2019) and Akcigit, Hanley and Stantcheva (2022).

[6] More specifically, their argument is that information technology makes production more scalable, but adopting it is costly. This incentivizes firms to specialize in a narrow set of sectors and expand production in their chosen sectors.

looking at the changes in the key moments due to each relevant parameter. The decomposition shows that higher patent trading efficiency and better patent protection can jointly explain 20% of the observed production scope decrease and 108% of the reallocation of R&D activities. The remaining part of specialization is primarily due to changes in the production cost structure. The increased efficiency and protection result in a 0.64 percent point increase in the annual growth rate, which makes them the main drivers of economic growth in the 1980s and 1990s.

Besides adjusting production scope, firms may also target their innovation to their production to improve matching between the two. One measure of the targeting behaviors of the innovation process is the share of basic research in total R&D spending. Since basic research is defined as "an activity aimed at acquiring new knowledge or understanding without specific immediate commercial application or use," higher basic research share implies less targeted innovation.[7] Using the Survey of Industrial Research and Development (SIRD), this paper finds that basic research's share increased in the period when firms' production scope narrowed, implying that firms' innovation activities became less targeted. To check whether the new hypothesis can explain this trend, the baseline model is extended to include two types of innovation, basic and applied research, that differ in R&D costs, the probability of matching a firm's own production scope, and the importance of their output. Similar decomposition exercises are undertaken for the extended model. The result shows that the changes in patent trading efficiency and protection can explain 105% of the increase in the basic research share. The intuition is that basic research benefits more from patent trading as its output is harder to be utilized by the firm's own production.

Finally, this study uses regional and sectoral differences in firms' exposure to the propatent policies to test whether the pro-patent policy reforms are causes of the contraction in firms' production scope and the reallocation of R&D activities. The fraction of lawsuits invalidating the patents involved in legal disputes varied much across the twelve regional circuit courts before the establishment of the CAFC in 1982, as pointed out by (Henry and Turner (2006) and Han (2018)). The establishment of the CAFC significantly lowered the regional invalidation rates and made them more uniform. So, regions with a higher invalidation rate before the CAFC experienced a larger increase in the strength of patent protection. Using a difference-in-difference (DiD) approach, it is found that firms in regions with a higher pre-CAFC invalidation rate decreased production scope more. Using a triple difference (DDD) approach with firm sizes being another dimension of the difference, it is found that small firms in regions with a higher pre-CAFC invalidation rate increased R&D intensity more, while large firms decreased it more. Furthermore, genetic engineering and software were two of the most controversial fields of patentability

[7] This is the definition of basic research in the Survey of Industrial Research and Development (SIRD).

in the 1970s. However, shortly before the establishment of the CAFC, the Supreme Court approved patentability in these two fields in two landmark cases, setting precedents for future cases. Therefore, these two fields experienced the most increase in patent protection strength and consistency in regional decisions. The share of firms' employment in these two fields before 1982 is used as a proxy for the exposure to the change in patent protection. With a Triple-Difference (DDD) approach, a finding is that firms with higher exposure were more likely to shrink production scope. These empirical results provide evidence of causality from the patent reforms to the two specialization patterns.

Related Literature

This paper is closely related to the literature on the impacts of patent trading and intellectual property rights (IPR) protection. The quantitative model in this study is based upon Akcigit, Celik and Greenwood (2016), which analyzes how the propinquity between the technology class of a firm's new patent and its past patents affects the value of the new patent to the firm and how a patent trading market shortens the propinquity. This paper extends this work in a variety of directions to address the newly observed specialization patterns. First, the paper introduces (endogenous) production scope and highlights that mismatches between innovation and production are critical to firms' boundary choices. The interaction between innovation and production scope decisions is new to the literature. Second, the paper introduces heterogeneity in firm production ability (reflected by size), which matters for the impact of patent trading. Production ability affects the expected value the firm can extract from new ideas through production and determines whether a firm benefits more from buying or selling patents. Third, the paper links patent trading to a wide range of changes in the 1980s and 1990s, e.g., production scope, reallocation and targeting behaviors of R&D. These linkages are novel. Other literature about the trading of knowledge (Eaton and Kortum (1996), Perla, Tonetti and Waugh (2021)) studies the impact of technology adoption on firms' innovation and growth but not on firms' boundaries. Most discussions about the influence of IPR protection focus on the trade-off between innovation incentives and inventors' monopoly power (Mukoyama (2003), Acemoglu and Akcigit (2012)). Some empirical studies suggest that the strength of the patent system facilitates the disintegration of the innovation industries by allowing trade in knowledge (Arora and Ceccagnoli (2006), Gans, Hsu and Stern (2008), Han, Liu and Tian (2020)).[8] However, as mentioned by Hall and Harhoff (2012), research in this area is still limited. There are few systematic theoretical and quantitative analyses about the role of IPR protection in firms' specialization decisions.

Theoretically, this paper contributes to the specialization literature by incorporating a new form of friction that determines firm boundaries between innovation and produc-

[8] A summary of the relationship between patents and innovation can be found in Moser (2013).

tion. According to Coase (1937), a comparison between market transaction costs and firms' internal organization costs determines the scope of a firm. The literature about specialization has studied various forms of external and internal costs. Williamson (1985) considers problems of incomplete contracts. Grossman and Hart (1986) and Costinot, Oldenski and Rauch (2011) emphasize the role of contractual frictions in determining firms' boundary.[9] Atalay, Hortaçsu and Syverson (2014) studies the determinants and effect of vertical integration and diversification. Grossman and Helpman (2002), Boehm and Oberfield (2020), and Bostanci (2021) discuss factors that affects firms' outsourcing decisions. Some papers (Chiu, Meh and Wright (2017), Baslandze (2016), Han (2018)) focus on frictions in the innovating sectors, but none of these papers considers how mismatches between innovation and production affect specialization.

Empirically, this research is related to the recent debates about US business dynamism. Hsieh and Rossi-Hansberg (2019) find that the gap between the number of industries of a top firm and that of an average firm is smaller in 2013 compared to 1977. They explain these changes by introducing a new technology that raises the fixed costs but lowers the marginal costs of production in the service industry. Related arguments about technological changes are in Aghion et al. (2019), De Ridder (2019) and Autor et al. (2020). Inspired by their research, the current study explores the specialization patterns more thoroughly by looking at the number of industries per firm for all years from 1978 to 2016. Findings are that all firms experienced a drop in the number of industries, and this drop was mostly driven by firms that performed R&D activities. The quantitative analysis of this paper supports the roles of both the increased tradability of intellectual properties and the change in the production cost structure. Besides, the observation of scope shrinkage with nearly constant average employment among the US firms in the 1980s and 1990s complements the findings that the aggregate concentration of the US firms was stable (White (2002)), but the within-industry concentration increased (Autor et al. (2020)).

This paper is also related to papers about growth slowdown after the 2000s (e.g., Akcigit and Ates (2019) and Olmstead-Rumsey (2019)) by explaining why there was high growth in the 1980s and 1990s. Consistent with a series of counterbalancing patent policies after 1999, the specialization patterns found in this paper also stabilized or reversed after the 2000s.[10] This suggests that specialization driven by patent protection and its impact on growth should be considered in making the optimal intellectual right protection policies, which is ignored in the current policy making process.

The rest of the paper is organized as follows. Section 2 presents the specialization

[9] A summary of the literature on firms' boundary can be found in Holmstrom and Roberts (1998).

[10] For example, the American Inventor Protection Act in 1999 required patent applications to be made public 18 months after being filed, regardless of whether patents were granted. This increased the risk of patent infringement. In 2006, Justice Kennedy of the US Supreme Court cast aspersions on business method patents, and the attitudes of the court system towards those patents became negative afterward.

patterns. Section 3 introduces the pro-patent policies. Section 4 shows evidence of a rising patent trading market and a declining matching rate between firms' innovations and production scope. Section 5 constructs an endogenous growth model with potential mismatches between innovation and production. Section 6 calibrates the model and evaluates the contribution of each possible explanation. Section 7 extends the model to include basic and applied research. Section 8 shows evidence of causality from the patent reforms to the specialization patterns. Section 9 concludes.

2 Specialization Patterns

This section exhibits the trends of production scope and R&D activities of US firms. The datasets involved are the Longitudinal Business Database (LBD) constructed by the US Census Bureau;[11] the Survey of Industrial Research and Development (SIRD) collected by the US Census Bureau and the National Science Foundation (NSF); the Patent Data Project (PDP) collected and cleaned by the NBER; the Compustat Historical Segments and Fundamentals Annual. Appendix A provides more details about the data.

The LBD covers the universe of business establishments with paid employees in the U.S. It has a consistent 6-digit NAICS code constructed by Fort, Klimek et al. (2016) for each establishment and each year. This study uses the firm ID variable that identifies the ownership of each establishment to aggregate the number of the 6-digit NAICS codes of each firm and defines it as the production scope of a firm. Information about firms' patenting activities comes from the PDP. It records all patents issued by the U.S. Patent and Trademark Office from 1976 to 2006. A firm is classified as an innovating firm if it has ever been granted a patent between 1976 and 2006.[12] The SIRD provides R&D information of a nationally representative sample of for-profit R&D-performing firms. Using the sample weights in the survey, the Census Bureau and the NSF calculate countrywide statistics each year and publish them on the Industrial Research and Development Information System (IRIS). Both the LBD and SIRD use the Census Bureau's Business Register (BR) as the primary input to its sampling frame. According to DeSalvo, Limehouse and Klimek (2016), a firm in the BR is defined as an economic unit comprising one or more establishments under common ownership or control. The Compustat datasets are used in robustness checks.

[11] Description of this dataset can be found in Jarmin and Miranda (2002).

[12] Although patenting is not the perfect measure of innovation activities, it is the best proxy in the data that covers all US firms.

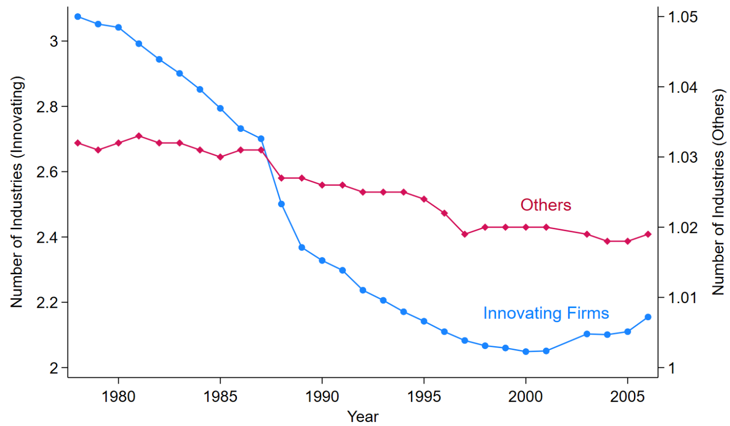


Figure 1: Trend of Production Scope by Innovating Activities

Notes: This figure shows the average number of 6-digit NAICS codes owned by US firms by year and innovating activities. The blue curve shows the trend for firms that have ever issued patents in the sample years; the red curve shows the trend of firms that have never issued patents.
Sources: Longitudinal Business Database (LBD); the Patent Data Project (PDP).

2.1 Production Scope

Since NAICS is constructed on a production-oriented framework and defines industries according to the similarity in the technology used to produce goods and services, the production scope captures the number of technologies a firm uses in production.[13] Figure 1 shows the average production scope of US firms with paid employees from 1978 to 2006 by whether they have ever issued a patent recorded by the PDP (innovating firms vs. others).[14] The scale for innovating firms is shown on the left y-axis, while the scale for other firms is shown on the right. Innovating firms produced in 3.07 6-digit NAICS industries on average at the beginning of the 1980s. This number experienced a sharp decrease by one-third to around 2.05 at the end of 1990s and then rebounded slightly after 2000. Other firms' production scope also decreased, but to a much lesser extent.[15] The finding that production scope of US firms decreased before the 2000s and increased afterwards is consistent with the trend discovered in Hoberg and Phillips (2022) using text-based analysis of firm 10-Ks, although their data starts in 1990 and ends in 2016.[16]

[13]NAICS is not market-oriented and thus does not capture the number of products produced by the same technology. For more information about NAICS, see `https://www.census.gov/naics/reference_files_tools/2022_NAICS_Manual.pdf`

[14]The data point for the year 2002 is omitted because, in the version of the LBD data available to the author of this paper, there is a problem in the scope statistics in 2002. Economists from the Census Bureau confirm that the newest version does not have the problem.

[15]Note that the average number of establishments per firm increased in the same period. So, the decrease in the number of industries was not due to firms having fewer establishments.

[16]Hoberg and Phillips (2022) focused on explaining the scope increase after the 2000s while this paper focuses on the scope decrease before the 2000s.

This paper does two checks. First, it looks at the trend of production scope with firm-size controlled and finds that innovating firms had a larger drop in scope than non-innovating firms of the same size.[17] Second, the paper deletes the auxiliary establishments (establishments that perform management and support services to other establishments) and repeats the exercises above. The results are very similar.[18]

2.2 Innovation Activities

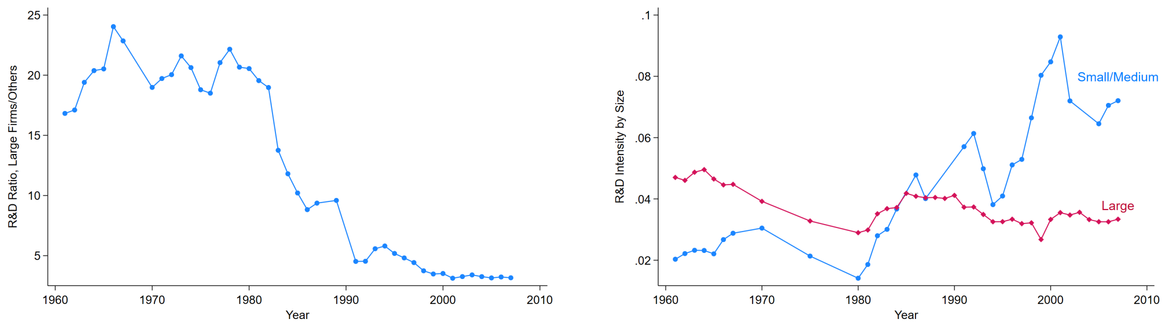
Figure 2a shows the ratio of total R&D spending by large firms to total R&D spending by small and medium firms. Here, a firm is regarded as small or medium if it has no more than 999 employees, while a large firm has at least 1000 employees. This ratio started to drop after the early 1980s and stabilized after 2000, indicating that US R&D activities have shifted from large to small and medium firms. To look at the intensive margin, Figure 2b displays the R&D intensity of US R&D performing firms by size. The R&D intensity is defined by the ratio of the aggregate R&D cost (excluding the federally funded part) of R&D performing firms to the net domestic sales of those firms. As shown in Figure 2b, the R&D expense-to-domestic sales ratio of small and medium firms started to surge after 1980, and the rising trend stopped after 2000.[19] In the same period, the ratio of large firms slightly decreased. These diverging trends suggest that small and medium firms became more focused on innovation, while large firms more focused on non-innovation activities. To address the potential misreporting problem of R&D expenses, this paper checks another measure—the ratio of the number of citation-weighted patents to the number of employees for large and small/medium firms with patents in the LBD. The trend is shown in Figure 7 of the Appendix B.2, and the implications are very similar. According to Baumol (2002) and Akcigit and Kerr (2018), small firms has a comparative advantage in creating new ideas, while large firms are better at exploiting values from innovations through production and commercialization. The two panels of Figure 2, therefore, suggest that firms spent more efforts on areas where they had comparative advantage.[20] This paper also looks at R&D intensity by firm age and finds that the diverging patterns are not as salient as the trends by size, showing that firm size is the main force behind the divergence in R&D intensity.

[17] Section B.1 of the Appendix describes the methods and plots the trend in Figure 6.

[18] Figures of production scope after deleting auxiliary establishments are available upon request.

[19] The increase in the R&D expense-to-domestic sales ratio was more salient for smaller firms (e.g., firms with less than 100 employees or less than 50 employees).

[20] In the following sections, this paper will call all the non-innovation activities as production. Therefore, production indicates all activities that are complementary to innovation.



(a) Total R&D Expense Ratio

(b) R&D Intensity by Size

Figure 2: Trends of R&D Activities

Notes: This figure shows R&D spending by firm size. Panel (a) displays the ratio of total R&D spending by large firms to total R&D spending by small and medium firms. Panel (b) displays the R&D-spending-to-domestic-sales ratios by firm size.
Source: Survey of Industrial Research and Development (SIRD).

2.2.1 Robustness Checks

The increase in R&D intensity among small firms may be attributed to the surge in venture capital (VC) activity during the 1990s, which heavily targeted small, private companies. Consequently, the shift in R&D activity across firm sizes may be more pronounced in sectors with high VC activity. To address this concern, this paper examines the R&D-to-sales ratio by sector and firm size distribution using Compustat data, which is less influenced by private equity investments such as VC. The analysis reveals that the shift in R&D from large to small firms is a widespread phenomenon across major industries and is primarily driven by firms smaller than the 25th percentile (157 employees in the dataset) of the size distribution. Further details of the robustness checks are provided in Appendix B.3.

3 Driving Forces

The two decades (the 1980s and 1990s) that witnessed the specialization wave described in the previous section also experienced important technological improvement and policy reforms in the United States. These changes have significantly affected the patent market.

3.1 Technological Improvement

The rise of the information technology enabled the USPTO to transit from a paper search system to an automated search system for US patents and trademarks in the 1980s. According to the USPTO report, before the transition, a searcher needed to “look at the daily updated Patent Locator to identify the Patent Search Room stack location(s) of the respective class(es)/subclass(es)" and then "remove all the paper copies of the patents in a class/subclass to be searched from the stack location and take them to a desk and look through them."[21] The availability of an electronic system largely facilitated the search process. Since potential buyers in patent trading needs to attain sufficient information about the focal patent, the transition of the search system reduced the frictions in patent trade.

3.2 Policy Reforms

In the 1970s, the innovation activities in the U.S. were thought to fall behind other industrialized countries (Meador (1992)), so a series of policies were adopted to stimulate innovation and boost economic growth. Besides introducing the R&D tax credits at the federal level in 1981, the US government adopted a series of pro-patent reforms starting at the beginning of the 1980s that strengthened the protection of intellectual property rights. The US legal environment towards patents became increasingly positive in the following two decades until some counterbalancing new policies came out at the end of the 1990s. This paper will describe two major pro-patent policies starting in the 1980s.[22]

Extension of Patentability to Genetic Engineering and Software. The US Supreme Court's decision in 1980 in the case between *Diamond* and *Chakrabarty* approved the patentability of genetically engineered bacteria. The 1981 decision in *Diamond v. Diehr* affirmed patent protection of software. Bioengineering and software became two heavily patented areas then. The overall patent applications and issuances both doubled between 1980 and 2000 after a long stable phase before 1980.

Creation of the Court of Appeals for the Federal Circuit. Before 1982, legal disputes of patents were heard at district courts or regional appellate courts, which did not have consistent enforcement of the patent law across regions. The establishment of the Court of Appeals for the Federal Circuit (CAFC) in 1982 provided centralized patent jurisdiction. More importantly, it largely decreased the patent invalidation rates in legal disputes (Henry and Turner (2006), Han (2018)). The fraction of lawsuits that invalidated the patents involved plummeted from around 55% to 28% after the change in the court system.[23] The legal disputes of patents usually arise because one party is not willing to pay for using the patents another party (the patent holder) created. The party that wants the patents then sues the patent holder by claiming its patents are invalid. The invalidation rates of the court therefore captures the probability that the plaintiff wins the case and uses the patent for free. A lower invalidation rate indicates the court has stronger

[21] The full version of the report, "Report to Congress on the Removal of Classified Paper From the USPTO's Public Search Facilities," can be found on the USPTO website.

[22] A thorough description of the policy changes can be found in Gallini (2002).

[23] The full trend of the invalidation rates is shown by Figure 10 in Appendix B.4.

protection toward the patent holders' benefit.[24]

4 Patent Trading and Innovation Mismatch

4.1 Patterns of Patent Trade

Following technological and policy changes, the patent trading market experienced rapid growth, signaling that innovations became more tradable. Using the Patent Assignment Dataset (PAD) in conjunction with the Longitudinal Business Database (LBD),[25] this paper calculates the fraction of patents granted to U.S. firms each year that were ever subsequently traded through sales or mergers and acquisitions (M&As). The analysis reveals an increase in this fraction from 29.16% in 1983 to 38.73% in 2000. When weighted by patent citations, the increase is even more pronounced, rising from 30.99% in 1983 to 45.9% in 2000. Figure 3a presents a decomposition of patent trades (not weighted by citations) by timing, showing that the majority of the increase is attributable to early transactions—further evidence of enhanced trading efficiency.[26]

The pattern of patent trade for all patents granted by the USPTO (not limited to those granted to U.S. firms) exhibits a similarly rising trend. The trends are also similar for the fraction of patents transacted through sales and through M&As respectively, with the volume of M&As consistently accounting for approximately one-tenth of that of sales. Further details are shown in Figure 11b in Appendix B.5. Besides patent transactions, patent licensing activities also ballooned after 1980, as indicated by the rising trends of licensing fees and royalties presented in Arora and Gambardella (2010). Therefore, the increase in patent transactions through sales adn M&A should be viewed as a lower bound of the estimation for the increase in trading activities of innovations. Regarding who traded the patents, the main argument of this paper is consistent with the finding by Akcigit and Ates (2019) that a larger share of patents were traded from small firms to large firms.

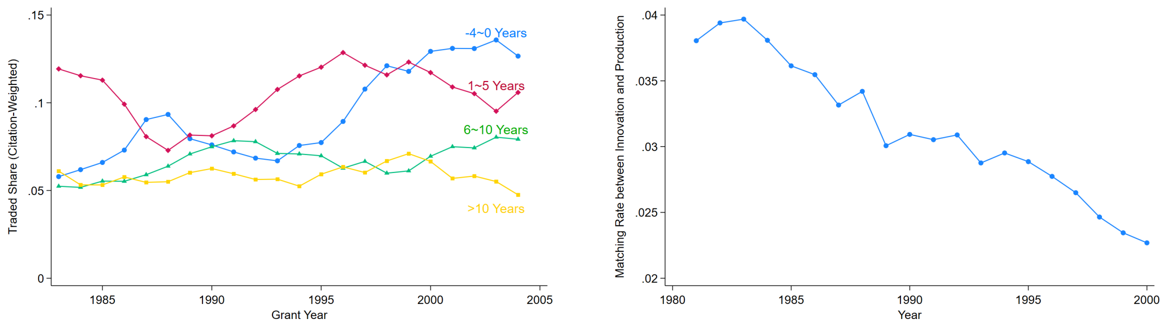
4.2 Patterns of Mismatch Between Innovation and Production

The growth of the patent trading market was accompanied by a declining trend in the matching rate between patents' technology classes and the production scope of their in-

[24] The ratio of the number of patent-related circuit court decisions to the number of patents-in-force have remained constant since 1980 (Marco et al. (2015)), showing that there was no clear change in the propensity of litigation (through circuit court decisions) after the reform.

[25] The PAD, collected by the USPTO, strives to maintain a comprehensive history of claimed interests in patents. For an introduction and statistical overview of this dataset, see Marco et al. (2015).

[26] Decompositions using citation-weighted patent trade fractions are presented in Figure 11a in Appendix B.5, which exhibit similar patterns.



(a) Fraction of Patents Involved in Trading

(b) Trend of the Matching Rate

Figure 3: Trading of Innovations and Matching with Production

Notes: This figure provides supporting evidence for the new hypothesis. Panel (a) illustrates the share of patents issued each year that were traded through sales or M&As within specific time windows: up to 4 years prior to issuance, 1–5 years post-issuance, 6–10 years post-issuance, and more than 10 years after issuance. Panel (b) displays the average likelihood that a firm's innovation output matches its production in each year.
Source: Patent Assignment Dataset (PAD); Longitudinal Business Database (LBD).

venting firms. The matching rate is defined as the ratio of the citation-weighted number of newly granted patents whose technology classes align with their inventors' industry classes to the citation-weighted total number of newly granted patents each year. This alignment between technology classes and industry classes is determined using the methodology in Silverman (2002) and a concordance linking SIC and NAICS codes. As shown in Figure 3b, in 1981, 3.8% of new patents fell inside of their inventing firms' production scope, while in 2000, the ratio decreased to 2.2%.[27] This trend implies that a firm's production has become less of a restriction to the usage of its innovations.[28]

The increased trading of innovations and decreased matching rate between a firm's innovation and production show that the market provides another channel for firms to monetize their R&D output.

4.3 Relationship between Patent Trade and Matching between Innovation and Production

To further investigate the relationship between patent transactions and the alignment of innovation with production, this paper estimates regressions using a dummy variable indicating whether a patent is transacted as the dependent variable. The key independent

[27] The unweighted ratio has the same trend and is available upon request.

[28] The decrease in the matching rate should not be due to changes in definitions of technology classes and industries over time versus the invariant concordance used. The concordance built by Silverman (2002) is based on the technology classes and industries in the early 1990s. So, if the invariant concordance used has any effect, we should predict the matching rate to be the highest in the early 1990s.

variable captures the likelihood that the patent falls within the production scope of its inventing firm. The regressions control for the patent's number of citations, its 4-digit IPC classification, and the firm's number of employees, according to Serrano (2010).

The results are presented in Table 1. Columns (1) and (2) document patent transactions up to the end of the data period, while Columns (3) and (4) focus on transactions occurring within five years of the patent's issuance. Columns (5) and (6) examine transactions within ten years of issuance. Additionally, Columns (2), (4), and (6) incorporate firm and year fixed effects. Across all specifications, the results consistently show a significantly negative relationship between the probability of being within the firm's production scope and the likelihood of patent trade. This indicates that patents falling outside a firm's production scope are more likely to be transacted. These findings hold not only in the cross-sectional analysis but also when controlling for firm and year variations.

Table 1: Patent Transactions and Innovation-Production Mismatch—Relationship

Dependent Variable	Ever Transacted		Transacted in 5 Years		Transacted in 10 Years	
	(1)	(2)	(3)	(4)	(5)	(6)
Within Scope	-0.0593**	-0.0890***	-0.0375*	-0.0625***	-0.0567**	-0.0878***
	(0.0290)	(0.0158)	(0.0201)	(0.0156)	(0.0287)	(0.0159)
Ln(Citations)	0.0178***	0.0186***	0.00807***	0.0120***	0.0185***	0.0188***
	(0.00132)	(0.000899)	(0.00204)	(0.00101)	(0.00133)	(0.000907)
Ln(Employment)	0.00501***	0.00492***	0.00362***	0.00304**	0.00510***	0.00510***
	(0.00127)	(0.00135)	(0.00129)	(0.00150)	(0.00126)	(0.00135)
4-digit-IPC fixed effect	YES	YES	YES	YES	YES	YES
Firm fixed effect	NO	YES	NO	YES	NO	YES
Year fixed effect	NO	YES	NO	YES	NO	YES
Observations	1,646,000	1,646,000	1,646,000	1,646,000	1,646,000	1,646,000
R-squared	0.028	0.332	0.021	0.298	0.028	0.330

Notes: The dependent variable is a binary indicator for whether a patent is transacted: (1) by the end of the data period, (2) within 5 years, or (3) within 10 years. "Within Scope" measures the likelihood that a patent's IPC aligns with its inventing firm's industries. All columns control the patent's 4-digit-IPC fixed effects. Columns (2), (4), and (6) further control firm and year fixed effects. Standard errors are clustered by the patent's 4-digit IPC code. To comply with Census Bureau disclosure requirements, the number of observations is rounded to the nearest thousand.
Source: Patent Assignment Dataset (PAD); Longitudinal Business Database (LBD).

5 Model

To explore the driving forces of the observed specialization phenomena and their effects on economic growth, a model is constructed in this section. In the model, there are potential mismatches between a firm's innovations and its production. Firms endogenously choose their production scope, R&D intensity, and whether to buy or sell innovation output on the patent market. The patent market is subject to search frictions, the efficiency of which and the bargaining power between buyers and sellers depend on the searching technology and legal environment towards patents. There are two types of production ability, which reflect firms' comparative advantage in innovation or production. Firms with a high production ability can extract higher value from new inventions through production and, on average, have larger size. Decisions of different types of firms are affected differently by patent trading, R&D tax credit rates, production cost structure, as well as the cost of new ideas.

5.1 Setup

There is a unit measure of firms in this economy, and each is exogenously and uniformly centered at a point on the industry circle shown in Figure 4. The industry circle contains all the industries in the economy and is assumed to have a radius of $\frac{1}{2\pi}$. At the beginning of each period, a firm chooses its production scope (ω)—the set of industries in which it will produce goods and services. Figure 4 shows an example of a firm that is centered at the top of the circle and chooses the arc ω as its production scope.[29] The absolute value of ω, $|\omega|$, stands for the number of industries the firm produces in and will be used in the following analysis. As the model only focuses on the symmetric equilibrium, the location of the center turns out to be irrelevant to firms' decisions.

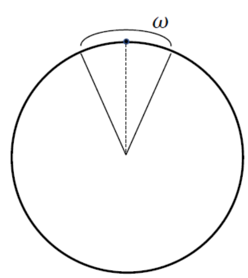


Figure 4: Schematic Diagram of the Industry Circle and Production Scope

Notes: This figure shows an example of a firm that is centered at the top of the industry circle and spans its scope symmetrically around its center.

A firm goes through two major stages of operation after the scope is determined: innovation and production.[30] The key assumptions of the model are twofold. First, the location of the innovation output cannot be entirely controlled by the firm, and therefore

[29]Whether the set of industries is connected is not assumed ex-ante, but will be solved from the model based on assumptions that will be unfolded later.

[30]The model can add a non-innovating sector whose productivity is dragged by the innovating sector, as what is done for the non-VC sector in Greenwood, Han and Sanchez (2022). The non-innovating sector captures firms that only adopt existing technology (they do not need to buy patents since most of the technology they use has passed the patent term.) The results in this paper will not change.

it may not necessarily fall inside the firm's production scope. Second, the firm cannot adjust its scope after the innovation stage and can only utilize the innovation output that matches its production scope. Between the two stages, firms can trade innovations on the patent market subject to a search and matching process. They can sell the innovation created by themselves and buy patents that match their production scope through an intermediary agent. A patent held by an agent persists over time with a probability of δ. This framework of trading patents via agents is consistent with the approach in Akcigit, Celik and Greenwood (2016) and closely reflects real-world practices. There are two exogenous changes in the search and matching process. (i) The matching efficiency increases. (ii) The invalidation rate of patents in legal disputes decreases, which, as will be shown later, is similar to a rise in the bargaining power of the patent sellers.

Each firm in this economy is characterized by production ability (m) and an innovation level (z). The production ability has two statuses, high (m_H) and low (m_L). The transition of statuses across periods is subject to a Markov process, $Q_{mm'} = \begin{bmatrix} q_{HH} & q_{HL} \\ q_{LH} & q_{LL} \end{bmatrix}$. In the stationary distribution, the shares of firms that have high and low production ability are respectively α_H and α_L. The innovation level is updated in each period according to the law of motion,

$$z' = z + \gamma(\mathbb{1}_{(RD\in\omega)}\mathbb{1}_k(m,z;\mathbf{z}) + \mathbb{B})\mathbf{z}, \tag{1}$$

where $\mathbb{1}_{(RD\in\omega)}$ is an indicator of whether the firm's innovation falls inside of its production scope. $\mathbb{1}_k(m,z;\mathbf{z})$ is an indicator that equals to 1 if the firm keeps its within-scope innovation. $\mathbb{B}$ is an indicator of whether the firm buys a patent that matches its scope. It is assumed that at most one idea can be implemented in each period, so $\mathbb{1}_{(RD\in\omega)}\mathbb{1}_k(m,z;\mathbf{z})$ and $\mathbb{B}$ are exclusive. γ is a constant lock-step growth of the innovation level. $\mathbf{z}$ is the employment-weighted average innovation level of the economy, defined by,

$$\mathbf{z} = \frac{\int\int mzdF(m,z;\mathbf{z})}{\alpha_H m_H + \alpha_L m_L}, \tag{2}$$

where $F(m,z;\mathbf{z})$ is the joint distribution of production ability and innovation levels among all firms at the end of the previous period.

The timing of events in each period is shown as follows:

m, z — Choose ω — $R\&D$ with i — $\mathbb{1}_{(RD\in\omega)}$ realizes — Search & trade ideas — z' realizes — Production

A firm starts a period with the newly realized production ability (m) and the innovation level (z) inherited from the end of the previous period. The value of the firm at this stage is denoted as $V(m, z; \mathbf{z})$. The firm chooses the production scope ω according to an increasing and convex management cost function in the number of industries,

$$C^e(\omega; \mathbf{z}) = \mu |\omega|^{1+\iota} \mathbf{z}^{\zeta/(\zeta+\lambda)} / (1+\iota),\ \iota > 0. \tag{3}$$

where μ and ι capture the shape of the cost function and are exogenous. ζ and λ are respectively the profit and labor share in the production function, as will be shown later.

After the scope is chosen, the firm begins to do R&D. This innovation process has a success rate of i, which is endogenously determined by the firm and also subject to an increasing and convex cost function,

$$C^i(i; \mathbf{z}) = \chi i^{1+\rho} \mathbf{z}^{\zeta/(\zeta+\lambda)} / (1+\rho),\ \rho > 0. \tag{4}$$

where χ and ρ capture the shape of the cost function and are exogenous. Both the management and innovation cost functions rise with the economy-wide innovation level, $\mathbf{z}$.

Whether the innovation process succeeds realizes then, together with the location of the output. The output is useful to the firm's own production only if it locates inside the scope. Firms that fail to innovate search for patent agents on the market as potential buyers. Firms that successfully innovate, but the innovation output is useless, sell the patent to an agent in a competitive market at the price $q(\mathbf{z})$, which is determined in the equilibrium. Simultaneously, they search for agents holding patents that align with their production scope. Firms that successfully innovate within their scope also have the option to sell their patents to agents. If they choose to do so, they can simultaneously act as buyers, searching for patents that complement their production needs. A patent agent can only process one patent at a time, and is searching for potential buyers. It is assumed that each patent agent and buyer have one unit of search effort. Agents spend their whole effort searching at the location of their patents; buyers evenly distribute their effort over their production scope. For any arc, d, on the industry circle, this paper denotes the total search effort on the arc by agents and by buyers respectively as $n_a(d)$ and $n_b(d)$. The total number of matches on the arc is

$$M(n_a(d), n_b(d)) = \phi n_a(d)^\nu n_b(d)^{1-\nu}, \tag{5}$$

where ϕ represents the matching efficiency, which is subject to exogenous changes. ν is the exponent. The odds of a successful match for an agent can be expressed as

$$s = \lim_{|d_0| \to 0} \phi \left(\frac{n_b(d_0)}{n_a(d_0)}\right)^{1-\nu}. \tag{6}$$

where d_0 is the neighborhood that spans symmetrically around the location of the seller's patent. The reason for taking the limit is that each agent is selling at only one point on the industry circle—the location of its patent. Since the model will only focus on the symmetric equilibrium, the location of the patent is not tracked. The odds of a successful match for a potential buyer depend on a function of the arc it searches over (its scope, ω),

$$b(\omega) = \phi\left(\frac{n_a(\omega)}{n_b(\omega)}\right)^{\nu}. \tag{7}$$

Finally, the new innovation level of the firm realizes according to the law of motion in (1). At the production stage, a firm maximizes its overall profit by choosing the optimal amount of capital and labor. The production function exhibits decreasing return-to-scale with regard to capital and labor. The profit, capital, and labor shares sum up to 1 ($\zeta + \eta + \lambda = 1$). Capital is hired at the rental rate $\tilde{r}$, and labor is hired at the wage rate w. It is assumed that goods in different industries are perfect substitutes and industries are symmetric. Denote the total capital and labor of the firm as k and l (the capital and labor in each industry is, therefore, $\frac{k}{|\omega|}$ and $\frac{l}{|\omega|}$). The firm's optimization problem is

$$\pi(\omega, m, z'; \mathbf{z}) = \max_{k,l} (mz')^{\zeta} k^{\eta} l^{\lambda} - \tilde{r}k - wl. \tag{8}$$

The production function suggests that firms with a higher production ability (m) get more profit at any given innovation level.

5.2 Consumer Preference

A representative household in this economy maximizes the lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t \frac{c(t)^{1-\epsilon}}{1-\epsilon}.$$

where $c(t)$ is consumption in period t, β is the discount rate of the future, and ϵ is the degree of risk aversion of the household. The household owns and rents capital to all the firms in this economy, which generates both a profit and a risk-free rate of capital return, $\frac{1}{r}$, in each period. The depreciation rate of capital is δ_c. So, the rental rate of capital, $\tilde{r}$, is $\frac{1}{r} - 1 + \delta_c$. The household also provides one unit of labor to firms, from which it earns a wage rate $w(t)$. The government levies a lump-sum tax, T, on the household to sponsor the R&D subsidy.

5.3 Firm Decisions

This section solves firms' decisions in backward order. At the final production stage, the first-order condition derives

$$k(\omega, m, z'; \mathbf{z}) = mz'(\frac{\eta}{\tilde{r}})^{1+\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}; \tag{9}$$

$$l(\omega, m, z'; \mathbf{z}) = mz'(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}}. \tag{10}$$

$$\pi(\omega, m, z'; \mathbf{z}) = mz'(1-\eta-\lambda)(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}. \tag{11}$$

The independence of total input and profit on the production scope implies that firms either span a wide range of industries but only touch on each of them, or focus on a narrow range of industries and deepen production in them. This independence is consistent with observations in the data, that US firms deepened production in fewer industries without changing much the total employment. The average employment of US firms was similar between the beginning of the 1980s and the end of the 1990s, even though the average number of industries was much lower at the latter period.[31]

The decision of R&D expenses is equivalent to determining the success rate (i) of innovation, as there is a one-to-one mapping between the two. Denote the value of a firm before the R&D decision as $D(\omega, m, z; \mathbf{z})$, taking the production scope as given. Then,

$$\begin{aligned} D(\omega, m, z; \mathbf{z}) = \max_i \{ & iX(\omega)\mathbb{1}_k(m, z; \mathbf{z}) \underbrace{[\pi(m, z'; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}')]}_{\text{Innovate and produce within } \omega} \\ & + (1 - iX(\omega)\mathbb{1}_k(m, z; \mathbf{z})) \underbrace{[b(\omega)(\pi(m, z'; \mathbf{z}) - p_b(m, z; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}'))]}_{\text{Buy an idea within } \omega} \\ & + (1 - iX(\omega)\mathbb{1}_k(m, z; \mathbf{z})) \underbrace{[(1 - b(\omega))(\pi(m, z; \mathbf{z}) + r\mathbb{E}V(m', z; \mathbf{z}'))]}_{\text{No idea within } \omega} \\ & + i(1 - X(\omega)\mathbb{1}_k(m, z; \mathbf{z})) \underbrace{q(\mathbf{z})}_{\text{Sell an idea}} - (1-\sigma)C^i(i; \mathbf{z}) \}, \end{aligned} \tag{12}$$

where the function $X(\omega)$ is the probability that the firm's innovation output falls inside its production scope, ω. It is assumed that (i) the closer an industry is to the firm's core business (center), the larger the probability the firm's inventions match that industry and generate value to the firm.[32] (ii) $X(|\omega|) = \xi|\omega|^{\psi}$ with $\xi > 0$ and $0 < \psi < 1$ if ω spans

[31] To be more specific, the average employment of US firms first decreased in the 1980s and then rebounded in the 1990s. The levels at the start and the end were similar.

[32] This assumption is supported by the empirical findings in Akcigit, Celik and Greenwood (2016) that the propinquity between a patent's technology class and the firm's main line of business positively affects the value of the patent to the firm.

symmetrically around the firm's center.[33] In the following analysis, $X(|\omega|)$ will denote the relationship between the within-scope probability and the length of the production arc, given that the arc is symmetric around the center.

$D(\omega, m, z; \mathbf{z})$ consists of five components, the first four of which describe the benefit of innovation in four different scenarios, while the last one of which is the innovation cost when the R&D tax credit rate equals to σ. The first scenario happens when the firm's innovation is successful, the output falls within the firms' production scope, and the firm keeps and uses the innovation in its own production. The firm then updates its innovation level according to the law of motion described in (1). $\pi(m, z'; \mathbf{z})$ is the profit in the current period with the updated innovation level (z'). $r\mathbb{E}V(m', z'; \mathbf{z}')$ is the discounted future value of the firm at the beginning of the next period. The second and third scenarios happen when the firm does not use its own R&D output, either because the innovation fails, or the innovation output does not match the firm's production scope, or a within-scope innovation is sold. The firm then searches on the patent market as a potential buyer. With probability $b(\omega)$, the firm matches with a patent agent. It buys the patent at a price $p_b(m, z; \mathbf{z})$ and updates its innovation level with the patent, as captured by the second scenario. With probability, $1 - b(\omega)$, the firm cannot find an agent, and therefore, its innovation level is not updated, as captured by the third scenario. The fourth scenario happens when the firm's R&D process succeeds, but the output falls outside the firm's own production scope, or a within-scope innovation is sold. In this case, the firm sells the patent to an agent. Once the patent is sold, the firm cannot use it any more. The decision of keeping or selling a within-scope innovation is made according to

$$\mathbb{1}_k = \begin{cases} 1 & \pi(m, z'; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}') \geq \begin{array}{l} b(\omega)\left(\pi(m, z'; \mathbf{z}) - p_b(m, z; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}')\right) \\ + (1 - b(\omega))\left(\pi(m, z; \mathbf{z}) + r\mathbb{E}V(m', z; \mathbf{z}')\right) + q(\mathbf{z}); \end{array} \\ 0 & \text{Otherwise.} \end{cases} \tag{13}$$

Denote the value of an agent under the average innovation level, $\mathbf{z}$, as $A(\mathbf{z})$. The determination of the buying price of a patent, also the transaction price, is through Nash bargaining, which can be described as follows,

$$\begin{aligned} p_b(m, z; \mathbf{z}) =& arg \max_{p_b} \left[p_b - r\delta A(\mathbf{z}')\right]^{\theta} [\pi(m, z'; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}') - p_b \\ & - (\pi(m, z; \mathbf{z}) + r\mathbb{E}V(m', z; \mathbf{z}'))]^{1-\theta}. \end{aligned} \tag{14}$$

The buyer and the agent choose the transaction price (p_b) to maximize the product of their

[33] As shown in Table 16 in Appendix D.2, the empirical estimation of $X(|\omega|)$ confirms this assumption.

surpluses. The agent's surplus is the transaction price minus the future value of retaining the patent. The buyer's surplus is the increase in the firm's value resulting from the updated innovation level, net of the payment, compared to its value at the original innovation level. θ represents the agent's bargaining power in the transaction. Importantly, the buyer's surplus, and consequently the transaction price, depends on the buyer's type.

Since agents cannot predict the type of buyers they will encounter, the price they offer to patent sellers, q, should be equal to the expected value of the patent. The distribution of types of potential buyers on the market is denoted as $G(m, z; \mathbf{z}')$ and will be determined endogenously in the equilibrium. The price, q, can be expressed as

$$A(\mathbf{z}) = s \int \int p_b(m, z; \mathbf{z}) dG(m, z; \mathbf{z}) + (1 - s) r\delta A(\mathbf{z}'). \tag{15}$$

The decision of the production scope at the beginning of each period is based on the tradeoff between the benefit and cost. The production scope, on the one hand, affects the ability that a firm monetizes its innovation output, and on the other hand, determines the management difficulty. The optimal scope solves,

$$V(m, z; \mathbf{z}) = \max_{\omega} D(\omega, m, z; \mathbf{z}) - C^e(\omega; \mathbf{z}), \tag{16}$$

where $C^e(\omega; \mathbf{z})$ is the management cost function as introduced in the model setup.

The government budget constraint can be expressed as the following,

$$T = \sigma \int \int C^i(i(\omega(m, z; \mathbf{z}), m, z; \mathbf{z}); \mathbf{z})) dF(m, z; \mathbf{z}). \tag{17}$$

5.4 Equilibrium

This paper focuses on a symmetric-balanced-growth-path (SBGP) equilibrium, where the employment-weighted average growth rate of the innovation level in the economy and the ratio of the average innovation level of firms with high production ability to that of firms with low production ability are constants. The variables in this equilibrium can be expressed as functions of the model parameters and are displayed in the following proposition. The proof is unfolded in Appendix C.1.

Proposition 5.1 (Symmetric Balanced Growth Path)**.** *There exists a symmetric balanced growth path of the following form:*
1. The employment-weighted growth rate of the aggregate innovation level, g, and the ratio of the average innovation level of firms with high production ability to that of firms with low production

ability, o, defined respectively by

$$g = \frac{\int\int m'z''dF(m',z')/\int\int m'dF(m',z')}{\int\int mz'dF(m,z)/\int\int mdF(m,z)};\ o = \frac{\int z'dF(m,z)|_{m=m_H}}{\int z'dF(m,z)|_{m=m_L}},$$

are constants.
2. The interest factor $r = \beta/g^{\epsilon\zeta/(\zeta+\lambda)}$; the rental rate on capital $\tilde{r} = g^{\epsilon\zeta/(\zeta+\lambda)}/\beta - 1 + \delta$.
3. The odds of a successful match for a potential buyer, $b(\omega)$, and for a potential seller, s, only depend on the total number of patent buyers and sellers, i.e., $b(\omega) = \phi(\frac{n_a}{n_b})^\nu$, $s = \phi(\frac{n_b}{n_a})^{1-\nu}$.
4. The production scope of each firm spans symmetrically around the center, and the length of the scope depends only on the production ability of the firm, i.e., $|\omega(m,z;\mathbf{z})| = \Omega(m)$.
5. The R&D success rate does not depend on the firm's innovation level, z, or the economy-wide innovation level, $\mathbf{z}$, i.e., $i(\omega,m,z;\mathbf{z}) = i(\omega,m)$.
6. The government budget constraint is,

$$T = \sigma(\alpha_H C^i(i(\Omega(m_H),m_H)) + \alpha_L C^i(i(\Omega(m_L),m_L))).$$

7. The value function $V(m,z;\mathbf{z})$ is linear in $\tilde{z}$ and $\tilde{\mathbf{z}}$, i.e., $V(m,z;\mathbf{z}) = v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}}$. The value of an agent is linear in $\tilde{\mathbf{z}}$, i.e., $A(\mathbf{z}) = a\tilde{\mathbf{z}}$. $\tilde{z} = z/\mathbf{z}^{\lambda/(\zeta+\lambda)}$ and $\tilde{\mathbf{z}} = \mathbf{z}^{\zeta/(\zeta+\lambda)}$.
8. Keeping or selling a within-scope innovation only depends on the firm's production ability, m.
9. The number of buyers of both types (n_{bH}, n_{bL}) and the number of agents (n_a) are

$$\begin{aligned} n_{bH} &= \alpha_H(1 - i^*(\omega^*(m_H),m_H)X(\omega^*(m_H))\mathbb{1}_k(m_H)); \\ n_{bL} &= \alpha_H(1 - i^*(\omega^*(m_L),m_L)X(\omega^*(m_L))\mathbb{1}_k(m_L)); \\ n_a &= \frac{\alpha_L i^*(\omega^*(m_H),m_H)(1 - X(\omega^*(m_H))\mathbb{1}_k(m_H)) + \alpha_L i^*(\omega^*(m_L),m_L)(1 - X(\omega^*(m_L))\mathbb{1}_k(m_L))}{1 - \delta(1-s)}. \end{aligned}$$

10. The buying price and the expected selling price of a patent is

$$\begin{aligned} p_b(m,z;\mathbf{z}) &= \theta(Jm + \frac{r}{g^{\lambda/(\lambda+\zeta)}}\mathbb{E}[v_1(m')|m])\gamma\tilde{\mathbf{z}} + (1-\theta)r\delta a g^{\frac{\zeta}{\zeta+\lambda}}\tilde{\mathbf{z}}; \\ q(\mathbf{z}) &= A(\mathbf{z}) = a\tilde{\mathbf{z}}, \end{aligned}$$

where both A and a are constants.

The intuition of the matching rate of a potential buyer only depending on the total number of buyers and agents is that firms are endowed with the same unit of search effort and have to dilute their effort at each point of the arc they search over. Therefore, although firms with different production scope have different chances of getting an in-scope idea if their innovation succeeds, they have equal opportunities to get an idea on the market. Besides, the matching rate of an agent is also the same, as on each point of the industry circle, there are equal number of buyers and agents.

The R&D success rate does not rely on individual and aggregate innovation levels because both the benefit and the cost of R&D depend only on the aggregate innovation level of the economy and the aggregate level cancels out in the calculation. The irrelevance of the R&D success rate with the innovation levels results in the production scope only relying on firms' production ability.

5.5 Relevant Parameters for Specialization

According to the analysis in the previous section, changes in the patent trading environment, the R&D tax credit rate, the production cost structure, and the difficulty in finding good ideas may be potential reasons for the observed specialization patterns. Parameters in the model that correspond to these changes are listed here.

The matching efficiency of the patent market, ϕ, reflects information frictions in the trading process. Technologies that reduce the search cost and policies that make inventions more visible on the market are predicted to raise the matching efficiency. The bargaining power of patent sellers, θ, reflecting protection towards patent holders, is directly related to the invalidation rate of patents. As shown in Section 3, the invalidation rate captures the probability that a buyer gets a patent for free through legal disputes. Denote the invalidation rate as f, then the transaction price is 0 with probability f. Therefore, the average buying price of patents is,

$$\begin{aligned} p_b(m,z;\mathbf{z}) =&\theta(1-f)[\pi(m,z';\mathbf{z})+r\mathbb{E}V(m',z';\mathbf{z}')-(\pi(m,z;\mathbf{z})+r\mathbb{E}V(m',z;\mathbf{z}'))] \\ &+(1-\theta)(1-f)r\delta A(\mathbf{z}'). \end{aligned} \tag{18}$$

A lower invalidation rate increases the buying price of patents, which is similar to higher bargaining power of the patent holder.

The R&D tax credit is directly captured by σ. A higher fixed cost of entering new industries corresponds to a larger scale and elasticity parameters in the management cost function (μ and ι in equation (3)). As the production function at the final stage is DRS to total production factors, and the total factors are the product of the number of industries and factors in each industry, decreasing the number of industries raises the marginal benefit of scaling production in each industry. It indirectly captures the decreasing marginal cost of production in an industry after entry, as proposed in the previous literature (Hsieh and Rossi-Hansberg (2019), etc.). Finally, the difficulty of finding good ideas is captured by the step size of innovations (γ) and the parameters (χ and ρ) in the R&D cost function.

6 Quantitative Analysis

The main goal of this section is to quantify the relative importance of the key drivers of the specialization patterns and their effects on economic growth. In particular, this study focuses on the four possible explanations: increased tradability of innovations (through both higher trading efficiency and better patent protection), the rise in the R&D tax credit rate, changes in the production cost structure, and changes in the difficulty of finding ideas. The quantitative analysis is undertaken in the following steps. First, the parameters in the model are set to fit data moments in the initial balanced growth path period, 1981-1985. This is the period of the paper search system used by the USPTO and the beginning of the policy reforms. Then, the relevant parameters as analyzed in Section 5.5 are changed to make the model fit the moments in the ending balanced growth path period, 1996-2000, with other parameters fixed in this process. This is the period when the electronic search system was widely used, occurring before the implementation of counterbalancing patent policies in the early 2000s. Untargeted moments are used to check the quality of the calibration. Finally, changes in firms' specialization decisions and the economic growth rate are decomposed into the contribution of each relevant explanation.

6.1 Calibration

There are eighteen parameters, $\{\eta, \lambda, \epsilon, \beta, \delta, \alpha_H, \alpha_L, \chi, \sigma, m_H, m_L, \nu, \gamma, \rho, \theta, \mu, \iota, \phi\}$, a transition matrix $Q_{mm'}$, and a function, $X(\omega)$, to be calibrated in the model. They are grouped into three categories. The first category comes from a priori information, as shown in Table 2. The capital and labor share (η and λ) are set respectively to be 0.28 and 0.57 (1/3 and 2/3 multiplied by a return to scale factor of 0.85). The profit share (ζ) is then 15%, which is consistent with the discussion in Guner, Ventura and Xu (2008). The degree of risk aversion for households (ϵ) is taken to be 2, a standard value in the literature. The discount factor (β) is set as 0.98, such that the interest rate of the model economy is 6%, a reasonable estimate for the long-run interest rate in the US. The depreciation rate of capital (δ) is chosen to be 0.07, consistent with the US National Income and Product Accounts. The paper defines firms of high production ability as those at the top 10% of the production ability distribution; firms of low production ability as the rest. This division is to make the two types of firms respectively represent the large and small firms defined earlier. Among all innovating firms between 1981 and 2000, around 9.1% are large firms (firms with more than 1000 employees). 55.1% of large firms turned out to be of high production ability, while only 5.5% of small and medium firms have high production ability. Therefore, in the following analysis, firms of high and low production ability largely correspond to large and small firms. The scale parameter in the R&D cost function (χ) is

Table 2: Parameter Values from a Priori Information

Parameter	Description	Value	Identification
η	Capital Share	0.25	Guner et al. (2008)
λ	Labor Share	0.60	Guner et al. (2008)
ϵ	CRRA Parameter	2.00	Standard
β	Discount Factor	0.98	Interest Rate
δ	Depreciation Rate	0.07	NIPA
α_H	Share of High Type	0.10	Imposed
α_L	Share of Low Type	0.90	Imposed
χ	R&D Cost, Scale	1.00	Normalization
σ	R&D Tax Credit Rate	0.05	Akcigit et al. (2018)

Notes: This table shows the parameter values adopted from a priori information. The division of firm types (α_H, α_L) to a large extent overlaps the division of firm size in Figure 2.

normalized to be 1, which is irrelevant to the quantitative results, as the calibrated step size of innovation (γ) will adjust to any changes in χ. The R&D tax credit rate (σ) is set at the effective level before 1980 as calculated by Akcigit, Ates and Impullitti (2018).

Parameters in the second category are pinned down by direct estimation from the data, as presented in Table 3. The sample used for estimation is all the firms in the Longitudinal Business Database (LBD) that have ever been granted a patent recorded in the Patent Data Project (PDP). Therefore, it is all the innovating firms. The sample spans from 1981 to 2000. Estimation of firms' production ability is based upon the solution of employment decisions in the model, $l(m, z') = mz'[(\alpha_h m_h + \alpha_l m_l)\mathbf{z}']^{-1}$. By taking the natural logarithm of both sides, it can be shown that the logarithm of a firm's employment equals the summation of the logarithm of its production ability, the logarithm of the innovation level, and aggregate factors. This study uses the accumulated citation-weighted patent stock as a proxy for a firm's innovation level and uses the time and industry fixed effects as proxies for the aggregate factors. Then, the firm's production ability is backed out from the residual term of the regression,

$$ln(emp_{ijt}) = \beta_1 \underbrace{ln(patentstock_{ijt})}_{ln(z')} + \beta_0 + u_t + v_j + \underbrace{residual_{ijt}}_{ln(m)}. \tag{19}$$

The production ability of the high type (m_H) and low type (m_L) are respectively estimated by the average production ability of firms at the top 10% and bottom 90% of the sample distribution. Once the production ability type is determined for each firm in each year, the four elements in the transition matrix ($Q_{mm'}$) are disciplined by the average fraction of firms transitioning from high to high, high to low, low to high, and low to low type between consecutive years.

The elasticity parameter (ν) in the matching function is estimated by running panel

Table 3: Parameter Values from Direct Estimations

Parameter	Description	Value	Identification
m_H	Prod. Ability of High Type	24.43	Regression
m_L	Prod. Ability of Low Type	0.70	Regression
$Q_{mm'}$	Type Transition Matrix	$\begin{bmatrix} 0.872 & 0.128 \\ 0.017 & 0.983 \end{bmatrix}$	MLE
ν	Matching Function, Exponent	0.70	Regression
$X(\omega)$	Within-scope Probability	$e^{-4.443} * \lvert\omega\rvert^{0.7643}$	Regression

Notes: This table shows the parameter values from direct estimations. The transition matrix reported is rounded to three decimal points to comply with the Census disclosure requirement.

regressions of the number of patent transactions on the number of potential sellers and potential buyers in different layers of industries (i.e., different numbers of digits of the NAICS code). Taking the natural logarithm of both sides of the matching function derives

$$ln(match\_num_{jt}) = \alpha_0 + \nu ln(seller\_num_{jt}) + (1-\nu) ln(buyer\_num_{jt}) + u_t + v_j + e_{jt},$$

where $seller\_num$ is the number of firms whose patent has a technology class that does not match any of the firm's 6-digit NAICS industries. $buyer\_sum$ is the number of firms that do not have an in-scope patent.[34] Whether the technology class of a patent matches the firm's industries is based on the concordance developed by Silverman (2002).[35] The results are shown in Table 15 in Appendix D.1. The value of ν is taken to be the average of the estimates.

The within-scope probability function ($X(\omega)$) is estimated as follows. Since it is optimal for firms to produce in industries close to its main line of business (center), this paper assumes all firms do so and only estimates the relationship between a patent's within-scope probability and the number of industries of its inventor. The function form is assumed to be $X(|\omega|) = \xi|\omega|^{\psi}$. This paper groups firms with patents in the LBD by the number of industries and regress the logarithm of the average fraction of patents that match their firms' production scope in each group on the logarithm of the industry number. ξ and ψ are estimated to be $e^{-4.443}$ and 0.7643.[36]

The third group of parameters is disciplined by minimizing the sum of squares of the distance between key moments in the data and the model-predicted values jointly in the initial balanced growth path (1981-1985). The growth rate in GDP per capita, after removing the cyclical components through the HP filter, is primarily affected by the

[34]The potential buyers may also include non-innovating firms. Including them in the regression will not change the results much.

[35]Silverman's concordance links the International Patent Classification (IPC) system to the U.S. Standard Industrial Classification (SIC) system. This study further links the SIC with the North American Industry Classification System (NAICS).

[36]The full regression results are shown in Table 16 of Appendix D.2.

Table 4: Parameter Values from the Minimum Distance Estimation

Parameter	Description	Value	Identification
γ	Step Size of Innovation	1.26	Growth Rate
$1+\rho$	R&D Cost Elasticity	1.56	R&D Cost/Sales
θ	Bargaining Power	0.20	Ratio (H and L)
μ	Management Cost, Scale	1.21E-4	Avg. Number of
$1+\iota$	Management Cost, Elasticity	2.34	Industries (H and L)
ϕ	Matching Function, Scale	0.04	Patent Traded Share

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments in the initial balanced growth path (1981-1985).

step size of growth driven by innovations (γ). The R&D cost-to-domestic sales ratio of innovating firms with high and low production ability are informative of both the elasticity of the R&D cost function ($1+\rho$) and the bargaining power of sellers on the patent transaction market (θ). The average industry numbers of innovating firms with high and low production ability are directly determined by the scale (μ) and elasticity ($1+\iota$) parameters in the management cost function. They are also indirectly influenced by sellers' bargaining power (θ). The citation-weighted share of patents traded within 10 years of issuance is linked with the scale parameter (ϕ) in the matching function. The estimated values of the relevant parameters are shown in Table 4. It is worth noting that both the R&D cost and management cost functions are convex, as assumed by the model, although no restrictions are imposed in the estimation process. The model predicted moments are almost the same as in the data, as shown by Table 5, attesting that the model fits the initial balanced growth path well.

Table 5: Model Fit for Key Moments in the Initial Balanced Growth Path

Targets	Data	Model
Economic Growth Rate (1981-1985)	2.13%	2.13%
R&D Cost/Sales of H Firms (1981-1985)	3.62%	3.62%
R&D Cost/Sales of L Firms (1981-1985)	2.83%	2.83%
Avg. Number of Industries of H Firms (1981-1985)	11.81	11.81
Avg. Number of Industries of L Firms (1981-1985)	1.92	1.92
The Share of Patents Traded (1983)	23.2%	23.2%

Notes: This table displays the targeted moments in the initial balanced growth path. The model and data moments are almost the same, showing the model fits the data well.

6.2 Recalibration to the Ending Balanced Growth Path

As pointed out in Section 5.5, the set of parameters, $\{\phi, \theta, \sigma, \mu, \iota, \gamma, \rho\}$, corresponds to the possible explanations for the specialization patterns. To match the ending balanced

Table 6: Model Fit for Key Moments in the Ending Balanced Growth Path

Targets	Data	Model
Economic Growth Rate (1996-2000)	2.22%	2.22%
R&D Cost/Sales of H Firms (1996-2000)	3.15%	3.15%
R&D Cost/Sales of L Firms (1996-2000)	6.71%	6.71%
Avg. Number of Industries of H Firms (1996-2000)	6.31	6.31
Avg. Number of Industries of L Firms (1996-2000)	1.61	1.61
The Share of Patents Traded (2000)	37.0%	37.0%

Notes: This table displays the targeted moments in the ending balanced growth path. The model and data moments are almost the same, showing the model fits the data well.

Table 7: Model Fit for Untargeted Moments

Moments	Data	Model
Within-scope Prob. of H Firms (1981-1985)	6.65%	7.76%
Within-scope Prob. of L Firms (1981-1985)	2.92%	1.94%
Within-scope Prob. of H Firms (1996-2000)	3.79%	4.81%
Within-scope Prob. of L Firms (1996-2000)	2.25%	1.69%

Notes: This table displays the untargeted moments. The model successfully captures the trend and magnitude of the within-scope probability of innovations for the two types of firms in the two periods.

growth path, this paper sets the new R&D tax credit rate as the actual effective rate, 24%, in the 1990s. Other parameters in this set are recalibrated to make the model fit the economic growth rate, the R&D cost-to-domestic sales ratio, the average industry numbers of innovating firms with high and low production ability, and the fraction of patents ever transacted in 1996-2000. The value of parameters out of this set is fixed in the recalibration process. The performance is displayed in Table 6, showing a good match between the model and data.

6.3 Untargeted Moments

To further check the quality of the calibration, this paper compares the model-predicted values with the real values of some untargeted moments. The within-scope probabilities for the two types of firms in the model ($X(\omega_H)$ and $X(\omega_L)$) are compared with the average matching rates between the firms' industry classes and their patents' technology classes. As shown in Table 7, they are very close in both periods. This suggests that parameters estimated from changes in production scope and innovation intensity successfully capture the declining matching rate between innovation and production.

Table 8: Changes of Parameter Values

	Old BGP	New BGP	Interpretation
ϕ	0.04	0.07	Matching efficiency increase
θ	0.20	0.29	Sellers' bargaining power increase
μ	1.21E-4	1.26E-4	Higher costs of large scope
$1+\iota$	2.34	3.04	More DRS to scope
γ	1.26	1.27	Fall in R&D efficiency
$1+\rho$	1.56	1.31	

Notes: This table compares the calibrated values of key parameters in the two balanced growth paths. The last column interprets the parameter value change between the two BGPs.

6.4 Changes in Key Parameter Values

Comparison between the initial and ending values of the parameters are displayed in Table 8. Although the direction of changes of these parameters is not restricted in the recalibration process, it turns out to be consistent with the original predictions. There is an increase in the matching efficiency, ϕ, of the patent market and the bargaining power, θ, of patent sellers, confirming decreasing market frictions and stronger protection towards patent holders. The scale and elasticity parameters in the management cost function (μ and ι) are larger, implying that the cost of producing in multiple industries is higher. The very slight change in γ and the decrease in ρ suggests that success of R&D depends more heavily on innovation investment, indicating a fall in R&D efficiency.[37] In both balanced growth paths, firms with both high and low production ability optimally choose to keep their within-scope innovation output.

6.5 Decomposition

To gauge the contribution of each possible explanation, this paper sets the parameters that govern each explanation at the ending balanced-growth-path value while others at the initial steady-state value. Hypothetical moments about specialization and economic growth are derived in each case. Then the paper compares the hypothetical moments with the moments in the initial balanced growth path. The difference between them measures the effect of each mechanism. The decomposition process uses the formula,

$$\frac{M_i(\Theta_{81-85}, \kappa_{96-00}) - M_i(\Theta_{81-85}, \kappa_{81-85})}{D_{i,96-00} - D_{i,81-85}}, \tag{20}$$

where M_i is the ith moment in the model and D_i is the corresponding value in the data. κ is the set of key parameters that correspond to each explanation. Θ represents all pa-

[37] The elasticity of the R&D success rate with respect to innovation investment can be expressed as $\frac{1}{1+\rho}$.

rameters in the model except for κ. This formula isolates the contributions of the key parameters.[38]

Table 9 presents the decomposition results. The first row displays the direction of changes in the data regarding the average production scope, the R&D intensity of firms with high and low production ability, the share of patents traded, and the economic growth rate. Starting from the second row, positive numbers mean the predicted change is consistent with the direction of the actual change; negative numbers mean otherwise. Numbers in the columns regarding scope, R&D intensity and patent trade captures the share of actual change explained by each channel. The column regarding growth captures the percentage point of the annual growth each channel is corresponding to. The direction of changes predicted jointly by higher matching efficiency and better patent protection, is consistent with the direction of all the real changes. Quantitatively, the new hypothesis can jointly explain 20% of the decrease in production scope of innovating firms; 229% of the decrease in R&D intensity for firms with high production ability and 108% of the increase in R&D intensity for firms with low production ability. It is responsible for the bulk of (90%) the rise in the trading share of patents and leads to a 0.64 percentage points increase in growth. This study lists the respective contribution of the matching efficiency and sellers' bargaining power, finding that the former is the main driving force. The R&D tax credit has little explanatory power for the specialization patterns but contributes to a higher growth rate. A significant portion of both specialization patterns can be attributed to changes in the production cost structure. However, these changes have minimal impact on patent trading activities and negatively contribute to economic growth. Increased difficulty in finding good ideas contributes to a large part of the decrease in firms' scope but is muted in explaining other dimensions of specialization. The subsections below will discuss the effects of each mechanism in detail.

6.5.1 Increased Tradability of Innovations

The effect of this mechanism on the specialization patterns is mostly driven by the rise in the matching efficiency of the patent trading market. Both the buyers and sellers get higher matching rates on the market. Higher chances of trading decrease R&D incentives for potential buyers while increase R&D incentives for potential sellers. Since firms with high production ability benefit more from buying patents on the market, they decrease R&D intensity. Firms with low production ability benefit more from selling patents to other firms. Therefore, they have more incentives to do R&D. The increasing matching

[38] Another decomposition method sets the parameters that govern each explanation at the initial balanced-growth-path value while others at the ending steady-state value. The hypothetical moments constructed in this way are compared with the data moments in the initial balanced growth path. The decomposition results are similar and are available upon request.

Table 9: Effects of Key Parameters

	Prod. Scope	R&D(H)	R&D(L)	Patent Trade	Growth
Data	-	-	+	+	+
Patent Market (ϕ, θ)	20%	229%	108%	90%	0.64pp
Efficiency (ϕ)	20%	223%	51%	99%	0.42pp
Bargaining Power (θ)	-10%	3%	44%	-5%	0.14pp
Tax Credit (σ)	11%	-80%	-13%	-7%	0.35pp
Production Cost (μ, ι)	59%	326%	44%	4%	-0.36pp
Rare Good Ideas (γ, ρ)	81%	-359%	-57%	7%	-0.03pp

Notes: The first row shows the actual direction of changes in the data. In the second to seventh rows, positive values indicate that the direction of changes due to the corresponding parameters is consistent with the actual direction. Columns 2-5 report the share of change explained by each channel, while Column 6 reports the percentage-point growth rate explained by each channel.

efficiency in patent trade makes the production scope less critical in determining the value of a firm's innovation output. So, there is a tendency to narrow the scope. The fraction of patents traded is directly linked to the matching efficiency, and therefore, is explained to a large extent.

The contribution of higher bargaining power mainly lies in the increase in the R&D intensity of firms with low production ability. This is because higher bargaining power increases the average transaction prices of patents. This increases the R&D incentives for firms with low-production ability, since they benefit more from selling patents.

Higher economic growth stems from two key factors. First, fewer ideas are wasted, as out-of-scope innovations can be leveraged through trade. Second, innovation activities are reallocated to firms with a comparative advantage, which arises from their identical innovation capabilities but relatively lower production ability.

6.5.2 R&D Tax Subsidy

An increase in the R&D tax credit enhances R&D intensity among firms with high production ability, albeit at the expense of a slight reduction in R&D intensity among firms with low production ability. High-production-ability firms benefit more because they can more effectively monetize innovation through their own production. The reduced R&D intensity among lower-production-ability firms arises from general equilibrium effects. Overall, higher R&D intensity significantly boosts economic growth.

6.5.3 Changes in the Production Cost Structure

Changes in the production cost structure contribute positively to both specialization patterns. A higher cost of producing in multiple industries directly shrinks firms' production scope. Smaller production scope reduces the likelihood of matches between innovation and production, thus disincentivizing firms to do R&D. This explains the decline in high-type firms' R&D intensity. The slight increase in low-type firms' R&D intensity is mainly due to the general-equilibrium effect. This mechanism alone has minimal effects on patent trading activities. It negatively affects growth as mismatches between innovation and production increase, and more inventions are wasted.

6.5.4 Good Ideas are Harder to Find

As innovation becomes more investment intensive, there is a direct decrease in the incentive to do R&D. Then, successful R&D output becomes scarcer and more valuable. So, firms with higher production ability (the ones that benefit more from R&D output) invest more in innovation. This predicts a shift of R&D activities to firms good at production, contradictory to the trend in the 1980s and 1990s. The decrease in firms' production scope is mostly driven by a significant decrease in the scope of firms with low production ability. This is because those firms sharply reduce their R&D effort and get lower benefits from expanding production scope. The change in the R&D cost function contributes negatively to growth as idea generation is more costly than before.

7 Discussion and Extension

Quantification of the baseline model shows that increased tradability of innovations can explain a sizable share of the decrease in production scope and the reallocation of R&D activities. However, this new hypothesis may be subject to several challenges. First, the direction of causality is not clear. It is possible that the more vibrant patent trading activities are the result of narrower production scope of firms, i.e., firms produce in fewer industries due to changes in the cost structure and then have to depend on the market for monetizing innovation as it becomes harder to match innovation output with their own production. Second, the potential mismatch between innovation output and production may not play an important role. Intellectual products may be similar to other goods in the sense that the inventing process requires ingredients from other intellectual properties. Alleviation of the incomplete contract problem in the ingredient trading process may also lead to more patent transactions and shrinkage in production scope.

To check whether the new hypothesis holds in front of these challenges, this paper looks at changes in the targeting behaviors of firms' R&D activities. If the reverse causality is true, it should be predicted that R&D becomes more targeted as the firm spans fewer industries. If there is no mismatch between innovation and production, but only the incomplete contract problem in the ingredient trading process for new inventions, the targeting behaviors of innovation will increase with patent trade since firms no longer need to invent every ingredient. On the contrary, the new hypothesis in this study predicts the R&D activities to be less targeted, as the type of R&D that is less likely to match the firm's own production benefits more from the trade of intellectual properties.

7.1 Data Patterns

The targeting behavior of the innovation process can be measured by the expense shares of different R&D types—basic research, applied research, and development. They differ in the probability of being applied to a specific production process.[39] This study uses the ratio of basic research to basic plus applied research expenses (the red curve) and the ratio of basic research to total R&D expenses (the blue curve) as proxies for firms' targeting behaviors in R&D. A higher share implies less targeting and broader R&D scope. Figure 5 shows the two ratios over the years.[40] They both picked up at the beginning of the 1980s, and the rising trends continued in the following two decades—the same period when the patent market grew. The pattern of widening R&D scope in the 1980s and 1990s is also supported by Akcigit and Ates (2019), in which the authors use the average length of patent claims as a measurement of the R&D scope. This pattern suggests that the reverse causality and the ingredient trading theory are insufficient to address the newly found specialization wave.

7.2 Model Extension

The baseline model is extended to study the impact of the new hypothesis on firms' targeting behaviors in the innovation process. Now, firms choose the success rates (equivalent to expense) of two types of research at the innovation stage—(a)pplied and (b)asic research. The two types of research differ in three dimensions: (i) the scale and elasticity parameters in the R&D cost function. (i.e., $\chi^b \neq \chi^a$, $\rho^b \neq \rho^a$), (ii) the probability of the innovation output falling inside the firm's own production scope (i.e., $X^b(.) \neq X^a(.)$), and (iii) the step size of successful inventions coming from basic research and from applied research (i.e., $\gamma^b \neq \gamma^a$). Each firm is endowed with two units of search effort—one for basic research output and the other for applied research output. The innovation level of

[39] In the Survey of Industrial Research and Development (SIRD), basic research is defined as "the activity aimed at acquiring new knowledge or understanding without specific immediate commercial application or use;" applied research is "the activity aimed at solving a specific problem or meeting a specific commercial objective;" development is "the systematic use of research and practical experience to produce new or significantly improved goods, services, or processes." Therefore, basic research has the broadest targets.

[40] Only data before 1998 is shown because statistics for 1998 and later years are not directly comparable to statistics for 1997 and earlier years, according to the statement made by the SIRD.

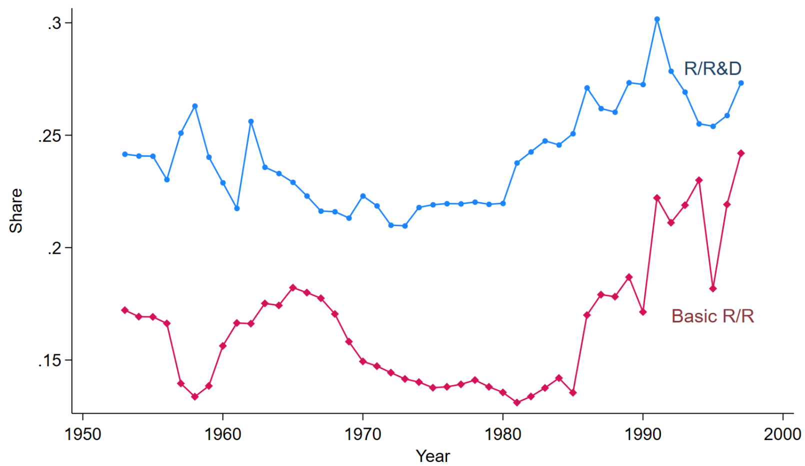


Figure 5: Share of Research Spending on Basic Research

Notes: This figure shows two measures of the basic research spending share. The red curve is the ratio of basic research expenses to total research expenses; the blue curve is the ratio of basic research expenses to total research and development expenses.

a firm is updated in each period according to the following law of motion,

$$z' = z + \sum_{j \in \{a,b\}} \gamma^j (\mathbb{1}^j_{(RD \in \omega)} \mathbb{1}^j_k(m, z; \mathbf{z}) + \mathbb{B}^j)\mathbf{z}, \tag{21}$$

where $\mathbb{1}^j_{(RD \in \omega)}$ is an indicator of whether the firm's type-j (applied or basic) research output falls inside its production scope. $\mathbb{1}^j_k(m, z; \mathbf{z})$ is an indicator that equals to 1 if the firm keeps its within-scope type-j innovation output. $\mathbb{B}^j$ is an indicator of whether the firm can buy a type-j (applied or basic) patent that matches its scope.

The new timeline is shown as follows.

m, z | $\mathbb{1}^a_{(RD \in \omega)}, \mathbb{1}^b_{(RD \in \omega)}$ | z' realizes

Choose ω | R&D with i_a, i_b | Search & trade ideas | Production

The following proposition holds. Characterization and proof of Proposition 7.1 are presented in Appendix C.2 and C.3.

Proposition 7.1 (Symmetric Balanced Growth Path)**.** *There exists a symmetric balanced growth path in the extended model.*

Table 10: Effects of Key Parameters

	Basic Research	Prod. Scope	R&D (H)	R&D (L)	Patent Trade	Growth
Data	+	-	-	+	+	+
Patent Market (ϕ, θ)	105%	24%	194%	96%	90%	0.61pp
Efficiency (ϕ)	67%	23%	190%	40%	97%	0.40pp
Bargaining Power (θ)	47%	-8%	0%	43%	-4%	0.14pp
Tax Credit (σ)	23%	10%	-72%	-11%	-5%	0.33pp
Production Cost (μ, ι)	16%	62%	262%	33%	3%	-0.32pp
Harder Ideas ($\{\gamma^j, \rho^j\}_{j\in\{a,b\}}$)	-54%	59%	-297%	-44%	8%	-0.39pp

Notes: The first row shows the actual direction of changes in the data. In the second to seventh rows, positive values indicate that the direction of changes due to the corresponding parameters is consistent with the actual direction.

7.3 Quantification of the Extended Model

Table 10 presents the explanatory power of the four mechanisms in the targeting behaviors of innovation and the other moments shown in the baseline calibration.[41] As shown by the first column, increased tradability of innovations is responsible for all (105%) of the increase in the share of basic research. The increase in R&D tax credit and changes in production cost structure also contribute to a small part of the increase. The increased difficulty of finding good ideas leads to a contraction in R&D scope. The impacts of the mechanisms on other moments are very similar to the results in the baseline model, confirming the robustness of the previous conclusions.

In sum, the rise in the share of basic research spending provides evidence of the important role of potential mismatches between innovation and production in explaining the observed specialization wave.

8 Empirical Analysis

This section empirically tests whether there is causality from the pro-patent reform to the specialization patterns. The main idea is to exploit the regional and sector differences in the exposure to policy changes and check whether they lead to different extents of the drop in scope and reallocation of innovation and production.

8.1 Institutional Background

The US federal court system has three main layers: district courts, circuit courts, and the Supreme Court of the United States. All patent-related cases are heard initially at one of

[41] The calibration process of the extended model is shown in Appendix D.4.

Table 11: Patent Invalidation Rates in District Courts under Different Circuit Courts

Circuit Court	Invalidation rate		Circuit Court	Invalidation rate	
	Before	After		Before	After
Boston	0.64	0.18	Chicago	0.54	0.30
New York	0.58	0.28	St.Louis	0.49	0.33
Philadelphia	0.74	0.32	San Francisco	0.51	0.29
Richmond	0.47	0.26	Denver	0.27	0.22
New Orleans	0.36	0.20	Atlanta	0.41	0.28
Cincinnati	0.60	0.30	DC	0.59	–

Notes: A higher invalidate rate before the establishment of CAFC means a more negative attitude towards patent holders. The circuit court of DC has too few observations after the CAFC era, so the invalidation rate is omitted.

the ninety-four district courts across the country. If there are challenges to the decisions, the case can be appealed to one of the circuit courts. Since the Supreme Court rarely hears patent-related cases, the circuit courts usually have the final say on those cases.

Before 1982, twelve circuit courts divided the country into different regions. Attitudes towards patents in the circuit courts had a significant discrepancy. Therefore, decisions of district courts under different circuit courts varied much in the first place. The second and fifth columns of Table 11 shows the fraction of lawsuits invalidating the involved patents in district courts of different regions from 1940 to September 1982. The legal environment towards patents was stable in this period.

In October 1982, Congress created the Court of Appeals for the Federal Circuit (CAFC). It has nationwide jurisdiction to hear appeals involving patent laws. So, decisions of district courts can be appealed to not only the twelve regional circuit courts but also the CAFC. The CAFC was more positive towards patents and had a much lower invalidation rate in its final decisions. Therefore, the decisions of district courts became lower and more uniform across different regions in the first place, as shown in the third and sixth columns of Table 11. Regions that had a higher patent invalidation rate before 1982 were more strongly affected by the CAFC.[42]

Precedents of court decisions in patent-related legal disputes often determine the patentability of similar objects afterward. Genetic engineering and software are two of the most controversial fields of patentability in the 1970s. In 1980, the Supreme Court ruled in the case between Diamond and Chakrabarty that genetically engineered bacteria involved in the case could be patented. This ruling was viewed as a turning point for the biotechnology industry in the following decades. In 1981, the decision of the Supreme Court in the dispute between Diamond and Diehr that software was not precluded from patentability also had a profound impact on court decisions afterward. These two land-

[42] Although there are forum shopping behaviors, firms are more likely to bring their lawsuits to the district court where they are located due to home-field advantage (Moore (2001)).

mark cases happened just before the establishment of the CAFC, making these two used-to-be controversial fields experience the most reduction of inconsistency among different regions. This leads to another dimension of difference in firms' exposure to policy shocks.

8.2 Estimation Strategy

The following Difference-in-Difference (DiD) regression explores whether regional differences in the change of patent protection led to different extents of contraction in firms' production scope,

$$ln(ind_{ist}) = \alpha_i + \beta * inval_{c,pre} * post_t + \gamma X_{ist} + \mu_t + \epsilon_{ist}, \tag{22}$$

where the dependent variable, ind_{ist}, is the number of 6-digit NAICS industries of the firm i in the LBD. s is the state of its headquarters before the year of the CAFC establishment. The headquarter is measured by the state where the firm has the most employment. t is the year of the observation. The main explanatory variable is an interaction between $inval_{c,pre}$, the patent invalidation rate of the circuit court, c, that the state, s, belongs to prior to the CAFC era, and a dummy variable, $post_t$, that indicates whether the year is before or after the establishment of the CAFC. The control variables, X_{ist}, include the log of firm's employment, the effective federal and state corporate income tax rates, and R&D tax credit rates calculated by Wilson (2009), and the log of state-level real GDP. Firm-fixed effects, α_i, and year-fixed effects, μ_t, are also included in the regression to exclude permanent cross-firm and time differences. The coefficient, β, captures the relationship between the different changes in firms' production scope and the different changes in patent protection strength across regions. A negative β implies firms in regions that experienced a larger increase in patent protection (i.e., a larger decrease in the invalidation rate) decreased production scope more.

Sectoral differences add another dimension of difference in the exposure to patent protection. The following Triple-Difference (DDD) regression tests whether firms with a higher exposure decreased production scope more,

$$\begin{aligned} ln(ind_{ist}) = &\alpha_i + \beta_1 * high\_treat_i * inval_{c,pre} * post_t + \beta_2 * inval_{c,pre} * post_t + \\ &\beta_3 * high\_treat_i * post_t + \gamma X_{ist} + \mu_t + \epsilon_{ist}, \end{aligned} \tag{23}$$

where $high\_treat_i$ is the firm's share of employment in the NAICS code 541710 (Research and Development in the Physical, Engineering, and Life Sciences)[43] and 511210 (Software Publishers) prior to the CAFC. The rest of the variables are the same as defined earlier.

[43] Bioengeering is embodied in this code.

The other interaction terms are omitted in the fixed effects. β_1 captures the differential impact of the change in patent protection for firms in the two most controversial industries versus others; β_2 shows whether the effect of the CAFC concentrates in the two industries or stretches to more general industries.

To check whether regional differences in the change of patent protection resulted in diverging trends of R&D activities by small and large firms, this paper designs the following regression,

$$\begin{aligned} RD\_to\_sales_{ist} = & \alpha_i + \beta_1 * small_i * inval_{c,pre} * post_t + \beta_2 * inval_{c,pre} * post_t + \\ & \beta_3 * small_i * post_t + \gamma X_{ist} + \mu_t + \epsilon_{ist}, \end{aligned} \tag{24}$$

where $RD\_to\_sales_{ist}$ is the the firm's R&D expenses to domestic sales ratio, measuring R&D intensity. $small_i$ is a dummy variable indicating whether the firm had less than 1000 employees prior to the CAFC. The rest of the variables are the same as defined earlier. β_2 captures the impact of the change in patent protection on large firms' R&D intensity; $\beta_1 + \beta_2$ captures the impact on small firms. A negative β_2 implies that large firms in regions experiencing a larger increase in patent protection (i.e., a greater decrease in the invalidation rate) decreased the R&D-to-sales ratio more. Conversely, a positive $\beta_1 + \beta_2$ implies that small firms in regions with a larger increase in patent protection increased the R&D-to-sales ratio more.

The standard errors are clustered at the circuit court region by the post dummy level in all specifications.

8.3 Sample Description

The sample of the regression analysis for production scope is the innovating firms in the LBD that existed before or in 1982, the year of the establishment of the CAFC. The sample of the regression analysis for R&D intensity is all the firms in the SIRD that existed before or in 1982. The requirement of existence before the reform is to avoid endogeneity issues induced by changes in firms' headquarters due to the policy change. To be representative for all the innovating firms, the R&D intensity regression is weighted by the sample weight assigned to each observation in the SIRD. The sample period for all regressions is from 1976 to 1989, 7 years before and after the reform.[44] Summary statistics of the main variables are presented in Table 20 in Appendix E.1. The number of observations and the common control variables in the two samples (weighted for the SIRD sample) are comparable in magnitude.

[44] 1976 is the earliest year of the LBD, so the longest period this study can explore before the establishment of the CAFC is seven years. This study also runs the same regressions on the samples of six years and five years before and after the reform. The results are very similar.

Table 12: DiD Regression Results on Production Scope

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
Invalidation Rate*Post	-0.0326**	-0.0326**	-0.0332**	-0.0281**
	(0.014)	(0.014)	(0.014)	(0.013)
Ln(Employment)	0.0888***	0.0899***	0.0893***	0.0894***
	(0.007)	(0.007)	(0.007)	(0.007)
Real GDP	NO	YES	NO	YES
Tax Rates	NO	YES	NO	YES
R&D Tax Credits	NO	YES	NO	YES
Post Dummy	YES	YES	NO	NO
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	268000	268000	268000	268000
R-squared	0.944	0.944	0.944	0.944

Notes: The dependent variable is the logarithm of the number of 6-digit NAICS codes owned by the firm. The four columns have different control variables. Standard errors are clustered by circuit court regions $\times$ the post dummy. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

8.4 Regression Results

Table 12 displays the regression results of Equation (22) that exploits regional differences on production scope. The first two columns insert the *post* dummy in the regression instead of the year-fixed effects; the last two columns control the year-fixed effects. Columns (2) and (4) control the state-level characteristics while columns (1) and (3) do not. In all of the columns, there are negative and significant coefficients of the interaction term, implying that firms located in regions with a larger change in patent protection strength experience a larger drop in production scope.

Table 13 displays estimation of Equation (23) that includes sectoral differences. The different controls across columns are the same as in Table 12. The negative and significant coefficient of the triple interaction term suggests that firms in the highly treated industries (bioengineering and software) are more affected by the CAFC. The coefficient of $Invalidation\ Rate * Post$ is still significantly negative, showing that the impact of the CAFC is not limited to the two highly treated industries.

The average magnitude of the interaction term coefficient (-0.032) in Table 12 suggests that the decrease in the patent invalidation rates ($55\% - 28\%$) resulted in 0.86% decrease in firms' production scope. The average sum of the triple and double interaction term coefficients in Table 13 (-0.16) suggests that for firms fully exposed to the bioengineering and software industries, the decrease in the patent invalidation rates ($55\% - 28\%$) resulted in 4.32% decrease in firms' production scope. Since the overall decrease of firms' production scope is 11.8% in the period of the regression sample, the invalidation rate

Table 13: DDD Regression Results on Production Scope

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
High_treat*Invalidation Rate*Post	-0.134*	-0.132*	-0.132*	-0.128*
	(0.069)	(0.069)	(0.069)	(0.069)
Invalidation Rate*Post	-0.0301**	-0.0301**	-0.0307**	-0.0257*
	(0.014)	(0.014)	(0.014)	(0.013)
High_treat*Post	0.0840**	0.0833*	0.0833**	0.0829*
	(0.040)	(0.041)	(0.040)	(0.040)
Ln(Employment)	0.0888***	0.0899***	0.0893***	0.0894***
	(0.007)	(0.007)	(0.007)	(0.007)
Real GDP	NO	YES	NO	YES
Tax Rates	NO	YES	NO	YES
R&D Tax Credits	NO	YES	NO	YES
Post Dummy	YES	YES	NO	NO
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	268000	268000	268000	268000
R-squared	0.944	0.944	0.944	0.944

Notes: The dependent variable is the logarithm of the number of 6-digit NAICS codes owned by the firm. The four columns have different control variables. Standard errors are clustered by circuit court regions $\times$ the post dummy. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

decrease alone can explain 7.3% of the scope shrinkage for general firms and 36.7% for firms in the bioengineering and software industries.

Table 14 displays estimation of Equation (24) that explores effect of the policy change on the R&D intensity of small and large firms. The different controls across columns are the same as in Table 12. The positive and significant coefficient of the triple interaction term suggests that small firms increases R&D intensity relative to large firms due to the establishment of the CAFC. The coefficient of $Invalidation\ Rate * Post$ is negative, although not significant, showing that the CAFC decreases the R&D intensity of large firms.

The average magnitude of the coefficient of $InvalidationRate * Post$ (-0.036) in Table 14 suggests that the decrease in the patent invalidation rates ($55\% - 28\%$) resulted in 0.97 percentage points decrease in large firms' R&D intensity. The average sum of the $Small * InvalidationRate * Post$ and $InvalidationRate * Post$ coefficients in Table 14 (0.21) suggests that the decrease in the patent invalidation rates resulted in 5.67 percentage points increase in small firms' R&D intensity. These numbers are comparable to the overall changes in the large and small firms' R&D intensity.

Placebo tests show there are no pre-trends for the observed regional and sectoral differences. Appendix E.2 describes details about the tests.

Table 14: DDD Regression Results on R&D Intensity

Dependent Variable	R&D Expenses to Domestic Sales Ratio			
	(1)	(2)	(3)	(4)
Small*Invalidation Rate*Post	0.266***	0.223**	0.267***	0.223**
	(0.091)	(0.105)	(0.093)	(0.105)
Invalidation Rate*Post	-0.0456	-0.0215	-0.0544	-0.0215
	(0.036)	(0.055)	(0.035)	(0.055)
Small*Post	-0.177***	-0.111*	-0.136**	-0.111*
	(0.057)	(0.066)	(0.058)	(0.066)
Ln(Employment)	-0.0049	-0.00189	-0.00121	-0.00189
	(0.024)	(0.022)	(0.022)	(0.022)
Real GDP	NO	YES	NO	YES
Tax Rates	NO	YES	NO	YES
R&D Tax Credits	NO	YES	NO	YES
Post Dummy	YES	YES	NO	NO
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations (Weighted)	220000	220000	220000	220000
R-squared	0.719	0.72	0.72	0.72

Notes: The dependent variable is the firm's R&D-expenses-to-domestic-sales ratio. The four columns have different control variables. Standard errors are clustered by circuit court regions $\times$ the post dummy. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

9 Conclusion

This study finds novel patterns of firm specialization in the 1980s and 1990s in the US Census data. (i) Firms, especially innovating ones, narrowed down their production scope. (ii) Innovation activities shifted from large to small firms.

A new hypothesis is proposed to explain the observed phenomena—higher patent trading efficiency and better patent protection increased the tradability of intellectual properties, making production scope less critical in determining the value of a firm's innovations. Three major conclusions can be drawn in this paper. First, increased tradability of innovations accounts for 20% of the production scope decrease and 108% of the reallocation of innovation activities. Second, increased tradability of innovations leads to a 0.64 percent point increase in growth rates. Third, there is evidence of causality from the pro-patent reforms to the two specialization patterns.

This paper also finds in the data that the R&D activities of US firms became less targeted in the 1980s and 1990s. Quantitative results of the extended model show that increased tradability of innovations can explain 105% of the decrease in R&D targeting.

Using the regional and sectoral differences in the exposure to patent policy changes in the early 1980s, this paper provides empirical support for causality from the pro-patent reform to contraction in firms' production scope and the shift of innovation activities.

The findings of this paper suggest that innovation and production become more separate when patent trade is more prevalent. A potential extension is to allow firms to endogenously choose their production ability at some costs. Mirroring the result that firms with high production ability choose to do less innovation, it is predicted that firms with high innovation levels will spend fewer resources improving their production ability. This may provide a new explanation for the phenomenon found in Pugsley, Sedlacek and Sterk (2019) that high-growth startups ("gazelles") have grown less rapidly in size since the mid-1980s. Another extension is to allow the product market to face monopoly power and firms to have entry-and-exit decisions. This helps to capture the distinct features between M&A and patent sales and the strategic behaviors of large firms as in Cunningham, Ederer and Ma (2021).

An important policy implication of this paper is that stronger intellectual property rights protection has an impact that is often neglected—reducing mismatches between innovation and production through a market approach. It spurs specialization and provides a strong engine for economic growth. Specialization resulting from patent trade should be considered when optimizing the IPR protection policies.

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Online Appendix

A Data Description

The data used in this paper includes the Longitudinal Business Database (LBD), the Patent Data Project (PDP), the Survey of Industrial Research and Development (SIRD), the Patent Assignment Dataset (PAD), the Compustat Historical Segments and Fundamentals Annual. This section provides details about the information of the datasets and the construction of key variables.

A.1 The Longitudinal Business Database (LBD)

The LBD is an establishment-level data of all US businesses with paid employees collected by the Census Bureau. The dataset assigns a 6-digit NAICS code to each establishment capturing its main production activities and a firm ID to all establishments belonging to the same firm. The 6-digit NAICS code is constructed by Fort, Klimek et al. (2016) and is consistent over the years.

Production Scope A firm's production scope is defined as the number of unique 6-digit NAICS codes of all the establishments belonging to the focal firm. The production scope by year from 1978 to 2006 is defined as the average production scope of each year. Although the average number of establishments per firm increases in the sample period, the average scope decreases, as shown in Figure 1.

Firm Size The LBD documents the number of employees each firm hires in each year. Firms are defined as being "large" if they have more than 1000 employees and as being "small" otherwise.

A.2 The Patent Data Project (PDP)

The PDP contains information of all utility patents issued between 1976 and 2006. For each patent, the data documents the grant year, the (truncated adjusted) citations it receives, and the assignees (entities that the patent is granted to). Using the the Business Dynamics Statistics of Patenting Firms (BDS-PF) patent assignee-FIRMID crosswalk from the Census, this paper matches patents in the PDP with the LBD, therefore, derives all patents in the US that were granted to employer businesses between 1978 and 2006.

Indicator of Innovating Firms This paper defines innovating firms as firms in the LBD that have ever issued patents between 1978 and 2006 and non-innovating firms otherwise. This indicator serves as a classification of firms' innovating activities in Figure 1.

Matching Rate between Innovation and Production For patents granted to a firm in the LBD, this paper identifies whether their technology classes match the industries of the firm using the concordance built by Silverman (2002) and the SIC-NAICS crosswalk from the Census. Silverman (2002) constructs a frequency distribution of the proportion of patents assigned to a 4-digit IPC code and to a Canadian 4-digit SIC code indicating the usage of the patents manually by the Canadian Patent Examiners. Then it links the Canadian SIC codes to the US 4-digit SIC codes. This paper further links the US SIC codes to the 6-digit NAICS codes. Therefore, there is a concordance between the patent technology classes and the industry usage. Using this concordance, the paper derives the average matching rate between patents' technology classes and their firms' industries by year, which is presented in Figure 3b.

A.3 The Survey of Industrial Research and Development (SIRD)

The SIRD is an annual sample survey conducted by the Census Bureau that targets all industrial companies with 5 or more employees that perform R&D in the US. It intends to represent all-for-profit R&D-performing companies, either publicly or privately held. The sample is selected from the Business Register (BR) that cover all US firms with paid employees. The key variables include the sample firms' R&D expenditure, domestic sales, total employment, character of R&D work (basic research, applied research, and development), etc. The macro- and industry-level data is available on the NCSES website.

Total R&D Expense Ratio This ratio is constructed by dividing the total R&D expenses of large firms to the expenses of small firms by year using the public macro-level data of the SIRD. The cutoff between large and small firms is 1000 employees. The trend of this variable is shown in Figure 2a.

R&D Intensity by Size For large and small firms respectively, this paper divides the total R&D expenses in each year by the total domestic sales of firms in that year. The construction of this variable is also using the public macro-level data. The trend of this variable is shown in Figure 2b.

Share of Research Spending on Basic Research This paper defines two measures of the basic research share. The first measure is the ratio of basic research expenses to the sum of basic research and applied research expenses; the second measure is the ratio of basic research expenses to the sum of the total R&D expenditure. The trend shown in Figure 5 is using the public macro-level data, while the targets in the calibration of the extended model is using the micro-level data from the Census Bureau.

A.4 The Patent Assignment Dataset (PAD)

The PAD records all the changes of claims made to the US patents from 1981 to 2020. The data is from the USPTO website and it documents the type of transactions, the transaction time, and the assignors' and the assignee's information. This paper keeps only the transaction types "patent sales" and "mergers and acquisitions". Merging the PAD with the PDP identifies the patents issued in each year that have ever been transacted. The transaction can happen any time after the patent application, which can be even before the patent is issued. Since the PDP stops in 2006, the merged data tracks the transaction history of all the patents issued in or before 2006. The sample is very slightly affected by the right-censuring issue, since most of the patents are transacted after application and within the first 15 years after issuance.

Fraction of Patents Involved in Trading This fraction is derived by dividing the number of patents issued in each year that have been transacted by the total number of patent issuances of the year. The trend is shown in Figure **??**.

A.5 The Compustat Historical Segments and Fundamentals Annual

The Compustat Fundamentals Annual contains information of all the publicly listed firms in the US. It records the number of employees, the primary industry (4-digit SIC code), and the R&D spending of each firm.

Indicator of Innovating Firms in Compustat In Compustat, innovating firms are defined as firms that have ever reported positive R&D spending or have patents granted between 1978 and 1997 and non-innovating firms otherwise.

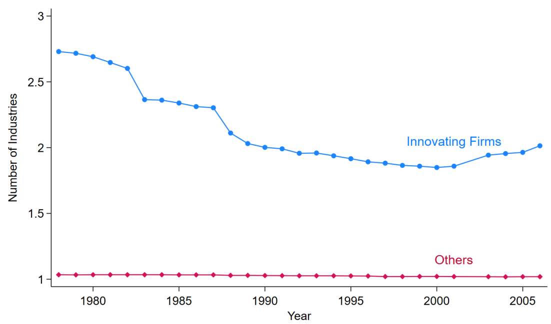
R&D Intensity by Size in Compustat The paper keeps the firms that have ever reported positive R&D spending or have patents granted between 1972 and 2006 to mimic the innovating firm sample in the SIRD. An innovating firm's R&D Intensity in Compustat is defined as the ratio between the R&D spending to the firm's total sales. The cutoff between the large and small firms are 1000 employees. The trends are shown in Figure 8.

Firms' Primary Industry in Compustat The primary industry of each firm in Compustat is based on the 4-digit SIC code assigned to each firm in the Fundamentals Annual. Manufacturing is corresponding to SIC codes 2000-3999; utility and transportation is corresponding to SIC codes 4000-4999; wholesale and retail trade is corresponding to SIC codes 5000-5999; services is corresponding to SIC codes 6000-8999.

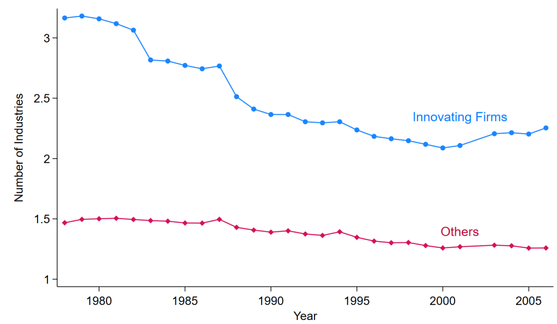
B More Empirical Evidence

B.1 Production Scope with Firm Size Controlled

To control the firm size, a regression of firms' production scope is run each year on a dummy variable of whether the firm is innovating or not, employment, and their interaction. Then the predicted production scope of innovating and other firms is calculated based on the estimated parameters when fixing the employment level at 20 and 1000, respectively. As shown in the two panels of Figure 6, at both employment levels, innovating firms shrank production scope more than other firms.



(a) 20 Employees



(b) 1000 Employees

Figure 6: Trends of Production Scope with Fixed Firm Size

Notes: This figure shows the average number of 6-digit NAICS codes owned by US firms when controlling firm size. This is created by running regressions of firms' production scope each year on a dummy variable of whether the firm is innovating or not, employment, and their interaction. Panel (a) shows predictions of a firm's production scope if the firm has 20 employees. Panel (b) shows predictions of a firm's production scope if the firm has 1000 employees.
Sources: Longitudinal Business Database (LBD); the Patent Data Project (PDP).

B.2 Another Measure of Innovation Intensity

Figure 7 shows the (citation-weighted) number of patents per employee for small/medium firms and large firms. They both increased starting from the early 1980s, but the increase was more salient for small/medium firms. The rising trends are partly due to the extension of patentability, but the different slopes of them reflect that small/medium firms engaged in more R&D activities.

B.3 R&D Intensity in Compustat

This paper checks the R&D by firm size in the Compustat data. The Compustat data covers all the publicly listed firms in the US, and therefore, is less affected by VC or other private equity investments. The Compustat Annual Fundamentals records the R&D spending and sales of each firm, thus, provides a direct measure of innovation activities.

Figure 8 shows the R&D expense to firms' total sales ratio in the Compustat Fundamental Annual by firm size. The threshold for distinguishing small and large firms remains at 1000 employees. The sample includes all the innovating firms in the Compustat. These firms either have positive R&D spending or have issued patents in the sample

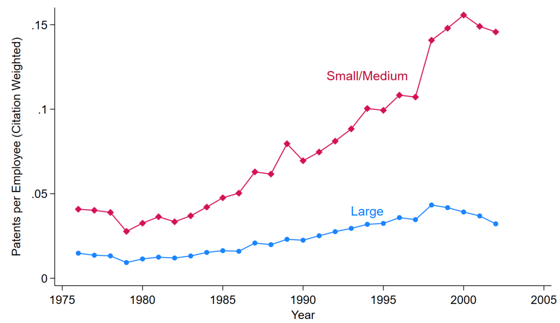


Figure 7: Patents per Employee by Size

Notes: This figure shows the ratio of the number of citation-weighted patents to the number of employees for large and small/medium firms with patents in the LBD. This provides another measure of R&D intensity by firm size that avoids the misreporting issue.
Sources: Longitudinal Business Database (LBD); the Patent Data Project (PDP).

period. This figure indicates that the shift of R&D from large to small firms is a general phenomenon for all major industries and is robust to private equity investment.

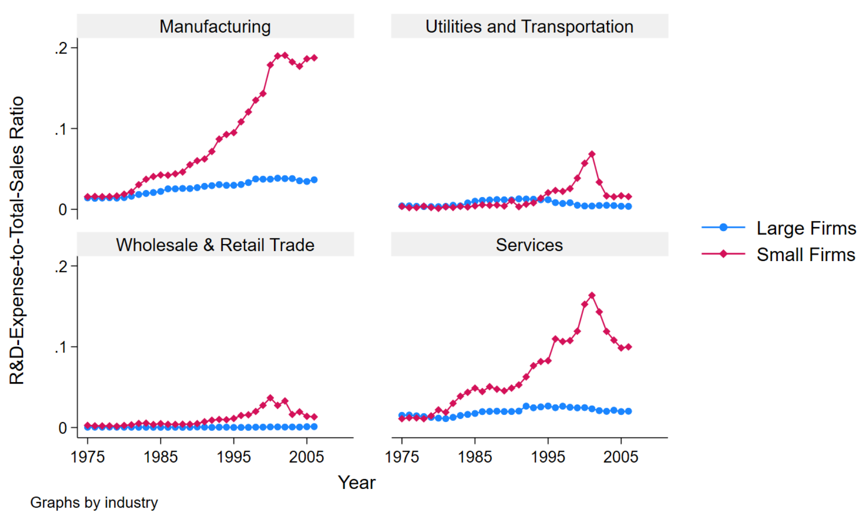


Figure 8: Trends of R&D Intensity by Firm Size and Industry

Notes: This figure shows the R&D expense to firms' total sales ratio in the Compustat Fundamentals Annual by year and the firm's main industry for innovating firms. The blue curve shows the trend for firms that have more than 1000 employees; the red curve shows the trend of other firms.
Sources: Compustat Fundamentals Annual.

To delve deeper into the relationship between R&D intensity and firm size, this study categorizes innovating firms' employment into six distinct brackets: below the 10th percentile; 10th to 25th percentile; 25th to 50th percentile; 50th to 75th percentile; 75th to 90th percentile; and above the 90th percentile. Specifically, these percentiles correspond to firms with approximately 36, 157, 958, 5311, and 22146 employees, respectively. Figure 5 presents the evolution of the R&D-to-total-sales ratio among innovating firms in Compustat over the years. Notably, the analysis reveals that the bulk of the increase in R&D intensity is attributed to firms falling below the 50th percentile, with smaller firms exhibiting a more pronounced uptick in this metric.

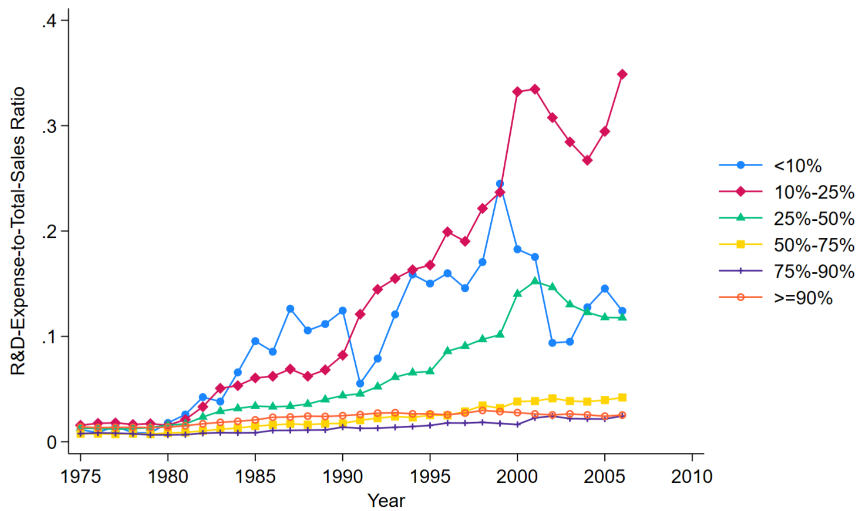


Figure 9: Trends of R&D Intensity across the Firm Size Distribution

Notes: This figure shows the R&D expense to firms' total sales ratio in the Compustat Fundamentals Annual by year and the firm's employment distribution. Firms' employment is segmented into six bins: below the 10th percentile; 10th to 25th percentile; 25th to 50th percentile; 50th to 75th percentile; 75th to 90th percentile; and above the 90th percentile. Specifically, the 10th, 25th, 50th, 75th, and 90th percentiles correspond to firms with approximately 36, 157, 958, 5311, and 22146 employees, respectively. *Sources:* Compustat Fundamentals Annual.

B.4 Patent Invalidation Rates

As shown by 10, the invalidation rates of patents in legal disputes experienced a sharp decrease after the establishment of the CAFC in 1982.

B.5 More Details on Patent Trade

Figure 11a shows the timing of the patent trade in Figure 3a. The blue, red, green, and yellow curves display, respectively the fraction of patents (citation-weighted) traded within four years before issuance, one to five years after issuance, six to ten years after issuance, and more than ten years after issuance. It should be noted that the descending trend of the yellow curve after 2000 is due to the right censoring issue. A comparison of the four curves suggests that most of the increase happened between 1980 and 2000, consistent with the timing of the pro-patent reforms; earlier transactions occurred more often,

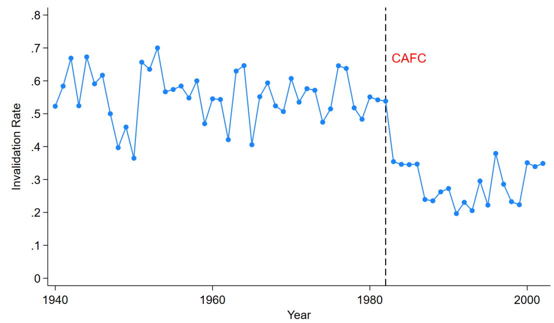
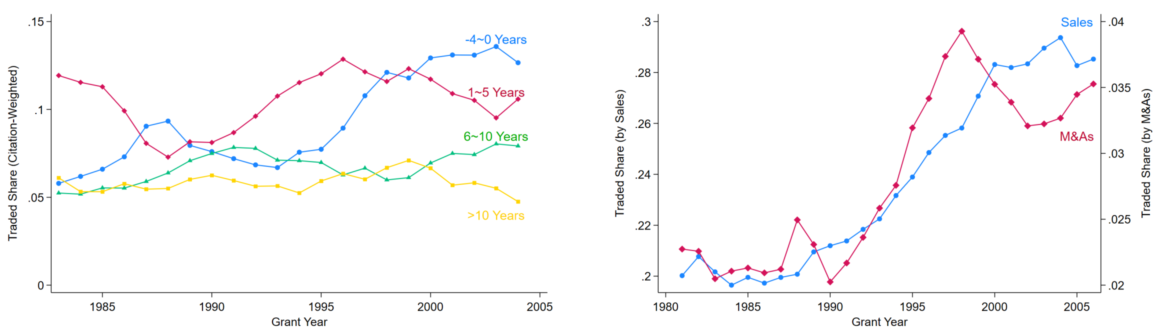


Figure 10: Patent Invalidation Rates in Lawsuits

Notes: This figure displays the average patent invalidation rates of the regional circuit courts by year. The red vertical line indicates the year of CAFC establishment.
Sources: Henry and Turner (2006)

evidence that the patent market has become more efficient.

Figure 11b illustrates the share of patents transacted through sales (blue curve) and M&As (red curve) respectively. Both types of transactions experienced an increase in the 1980s and 1990s, with the volume of the latter type being around one-tenth of the volume of the former. This figure also functions as a robustness check for Figure **??** in Section 4, as it encompasses all transactions (not exclusively those to US firms) and is devoid of weighting by patent citations.



(a) Patent Trade by Gaps from the Grant Year (b) Patent Trade by Transaction Type

Figure 11: Patent Trade by Different Classifications

Notes: Panel (a) displays the fraction of patents traded at different time windows. The fractions are weighted by the number of patent citations. Panel (b) displays the share of patents traded through sales and the share traded through M&As for all patents granted by USPTO. The scale for sales is shown the left y-axis, and the scale for M&As is shown on the right y-axis. The shares are devoid of weighting by patent citations.
Sources: Patent Assignment Dataset (PAD); Longitudinal Business Database (LBD).

C Proof of the Theory

C.1 Proof of Proposition 5.1

Proof. Denote the distribution of production ability and innovation levels among all firms at the end of the current period as $F(m, z'; \mathbf{z})$. Equation (10) implies that the labor market clearing condition can be written as

$$(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}} \int\int mz'dF(m, z'; \mathbf{z}) = 1. \tag{25}$$

Equation (25) can be transformed to

$$(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}}(\alpha_H m_H \mathbf{z}'_H + \alpha_L m_L \mathbf{z}'_L) = 1, \tag{26}$$

where $\mathbf{z}'_H$ and $\mathbf{z}'_L$ are, respectively, the average innovation level of firms with high and low production ability at the end of this period. They are defined by

$$\mathbf{z}'_H = \frac{1}{\alpha_H}\int z'dF(m_H, z'; \mathbf{z}); \tag{27}$$

$$\mathbf{z}'_L = \frac{1}{\alpha_L}\int z'dF(m_L, z'; \mathbf{z}). \tag{28}$$

The economy-wide average innovation level at the end of the previous period, $\mathbf{z}$, can then be expressed as

$$\mathbf{z} = \frac{\alpha_H m_H \mathbf{z}_H + \alpha_L m_L \mathbf{z}_L}{\alpha_H m_H + \alpha_L m_L}. \tag{29}$$

Assume $\mathbf{z}$ grows at a constant rate, g, across periods. Then, the labor market clearing condition can be further transformed to

$$(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}}(\alpha_H m_H + \alpha_L m_L)g\mathbf{z} = 1. \tag{30}$$

The wage rate, w, can then be expressed as

$$w = \lambda(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta+\lambda}}[(\alpha_H m_H + \alpha_L m_L)g\mathbf{z}]^{\frac{\zeta}{\zeta+\lambda}}, \tag{31}$$

which implies that it grows at a rate of $g^{\frac{\zeta}{\zeta+\lambda}}$. The total output and capital of the economy also grow at $g^{\frac{\zeta}{\zeta+\lambda}}$, since

$$\int\int Y(m,z';\mathbf{z})dF(m,z';\mathbf{z}) = (\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}(\alpha_H m_H + \alpha_L m_L)g\mathbf{z}; \tag{32}$$

$$\int\int K(m,z';\mathbf{z})dF(m,z';\mathbf{z}) = (\frac{\eta}{\tilde{r}})^{1+\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}(\alpha_H m_H + \alpha_L m_L)g\mathbf{z}, \tag{33}$$

where w grows at the rate $g^{\frac{\zeta}{\zeta+\lambda}}$, $\mathbf{z}$ grows at the rate, g, and all the other parameters are fixed.

A firm with production ability m and an innovation level z at the beginning of the period may or may not update its innovation level through R&D or trade. If it updates the innovation level, the profit of the current period is

$$\pi(m,z';\mathbf{z}) = \zeta m(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}(z+\gamma\mathbf{z}). \tag{34}$$

Otherwise, the profit is

$$\pi(m,z;\mathbf{z}) = \zeta m(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}z. \tag{35}$$

Denote $\tilde{z} = \frac{z}{\mathbf{z}^{\frac{\lambda}{\zeta+\lambda}}}$, $\tilde{\mathbf{z}} = \frac{\mathbf{z}}{\mathbf{z}^{\frac{\lambda}{\zeta+\lambda}}}$. Plugging the expression of w in (31) into (34) and (35) derives

$$\pi(m,z';\mathbf{z}) = Jm(\tilde{z}+\gamma\tilde{\mathbf{z}}),\ \pi(m,z;\mathbf{z}) = Jm\tilde{z}, \tag{36}$$

where $J = \zeta(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta+\lambda}}[(\alpha_H m_H + \alpha_L m_L)g]^{-\frac{\lambda}{\zeta+\lambda}}$. So, the difference of firm profit with the updated and non-updated innovation levels is $Jm\gamma\tilde{\mathbf{z}}$, which is not a function of the firm's current innovation level, z.

Next, a guess-and-verify procedure is used to derive the value of the firm at the beginning of the period, $V(m,z;\mathbf{z})$, and the value of the patent agent, $A(\mathbf{z})$. Conjecture

$$V(m,z;\mathbf{z}) = v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}};\ A(\mathbf{z}) = a\tilde{\mathbf{z}}. \tag{37}$$

Then, the surplus of the firm if being a buyer in the Nash bargaining problem (14) is

$$[\pi(m,z';\mathbf{z}) + r\mathbb{E}V(m',z';\mathbf{z}')] - [\pi(m,z;\mathbf{z}) + r\mathbb{E}V(m',z;\mathbf{z}')] = [Jm + r\mathbb{E}(v_1(m'))g^{-\frac{\lambda}{\zeta+\lambda}}]\gamma\tilde{\mathbf{z}}, \tag{38}$$

which is not a function of the firm's innovation level, z, either. Denote this surplus as $\Delta\psi(m;\mathbf{z})$ and use $B(m)$ as an abbreviation for $[Jm + r\mathbb{E}(v_1(m'))g^{-\frac{\lambda}{\zeta+\lambda}}]$. We have

$$\Delta\psi(m;\mathbf{z}) = B(m)\gamma\tilde{\mathbf{z}}. \tag{39}$$

The price this firm has to pay to buy a patent can be expressed as (Point 10)

$$p_b(m;\mathbf{z}) = \theta\Delta\psi(m;\mathbf{z}) + (1-\theta)r\delta a\tilde{\mathbf{z}}' = \theta B(m)\gamma\tilde{\mathbf{z}} + (1-\theta)r\delta a g^{\frac{\zeta}{\zeta+\lambda}}\tilde{\mathbf{z}}, \tag{40}$$

i.e., the buying price is the bargaining power of the seller times the trading surplus of the buyer. It only depends on the production ability of the buyer and the aggregate innovation level. By the zero-profit condition of the patent agents, the expected price a firm gets if selling a patent on the market is equal to the value of an agent, i.e., $q(\mathbf{z}) = a\tilde{\mathbf{z}}$. The selling price depends on the shares of searching effort from high-type buyers and low-type buyers. Since we focus on a symmetric equilibrium, the shares are constants on any arc of the technology circle, i.e.,

$$\frac{n_{bH}(d)}{n_b(d)} = \frac{n_{bH}}{n_b}, \ \forall d, \tag{41}$$

where $\frac{n_{bH}}{n_b}$ and $\frac{n_{bL}}{n_b}$ are the share of potential buyers with high and low production ability.

To solve firms' optimal innovation intensity and the "keep or sell" decision, it is necessary to derive the expressions of s and $b(\omega)$ in problem (12). Consider any arc on the circle. Without loss of generality, Figure 12 shows an arc d with length $|d|$. The total search effort by potential sellers on d equals to the number of potential sellers that have a patent located inside d. On a symmetric balanced growth path, sellers' patents are evenly distributed on the circle. So, $n_a(d) = |d|n_a$.

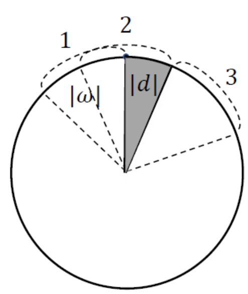


Figure 12: Schematic Diagram

Potential buyers that spend effort searching on d may have various scope. I classify these buyers according to the length of their scope. For potential buyers with scope length equal to $|\omega|$, their locations may span from 1 to 3. Buyers at location 1 or 3 spend measure 0 of search effort on d, while buyers at location 2 spend measure $\frac{|d|}{|\omega|}$ of search effort on d.

The total measure of search effort on d conditional on the buyer having $|\omega|$ as the scope length is an integral of effort from location 1 to 3, which can be expressed as

$$\int_0^{|d|} \frac{i}{|\omega|} di + \int_{|d|}^{|\omega|} \frac{|d|}{|\omega|} di + \int_{|\omega|}^{|d|+|\omega|} \frac{|d| + |\omega| - i}{|\omega|} di = |d|, \ \forall |\omega|, |d|. \tag{42}$$

This conditional measure does not rely on the scope length. So, the unconditional total measure of search effort on d is d times the total number of potential buyers, i.e., $n_b(d) = |d| n_b$.
The number of matches on the arc d equals to

$$M(s(d), b(d)) = |d| \phi n_a^{\nu} n_b^{1-\nu}, \forall d. \tag{43}$$

Potential buyers with scope ω will only search within its scope, so, the probability of meeting a seller is

$$b(\omega) = \frac{M(n_a(\omega), n_b(\omega))}{n_b(\omega)} = \phi (\frac{n_a}{n_b})^{\nu} \equiv b, \tag{44}$$

which is a constant and does not depend on the scope of the buyer. The probability for a potential seller to meet a buyer is

$$s = \lim_{|d_0| \to 0} \frac{M(n_a(d_0), n_b(d_0))}{n_a(d_0)} = \phi (\frac{n_b}{n_a})^{1-\nu}, \tag{45}$$

which is also a constant (Point 3).

Firms keep within-scope innovations if and only if doing so yields greater value than selling them (Equation (13)). Plugging the expressions derived above into Equation (13) shows that

$$\mathbb{1}_k = \begin{cases} 1 & [1 - b(1-\theta)] B(m) \gamma + b(1-\theta) r \delta a g^{\frac{\zeta}{\zeta+\lambda}} \geq a \\ 0 & \text{Otherwise,} \end{cases} \tag{46}$$

which only depends on the production ability, m, of the firm (Point 8).
Plugging the matching probabilities $b(\omega)$ and s into problem (12) derives the solution of firms' R&D success rate.

$$\begin{aligned} i^*(\omega, m) = &\{ \frac{1}{(1-\sigma)\chi} [X(\omega) \mathbb{1}_k(m) ((1 - (1-\theta) b) B(m) \gamma + b(1-\theta) r \delta a g^{\frac{\zeta}{\zeta+\lambda}}) \\ &+ (1 - X(\omega) \mathbb{1}_k(m)) a] \}^{\frac{1}{\rho}}, \end{aligned} \tag{47}$$

which only depends on the firm's production scope and production ability (Point 5).

The firm's value at the innovation stage, $D(\omega, m, z; \mathbf{z})$ is then

$$D(\omega, m, z; \mathbf{z}) = B(m)\tilde{z} + [\frac{(1-\sigma)\rho}{1+\rho}\chi i^*(\omega, m)^{1+\rho} + b(1-\theta)(B(m)\gamma - r\delta a g^{\frac{\zeta}{\zeta+\lambda}}) \tag{48}$$

$$+ r\mathbb{E}v_2(m')g^{\frac{\zeta}{\zeta+\lambda}}]\tilde{\mathbf{z}}. \tag{49}$$

If $\mathbb{1}_k(m) = 1$, $D(\omega, m, z; \mathbf{z})$ increases in $X(\omega)$, and therefore, is larger when ω is closer to the firm's center for any given length of ω. Consequently, firms always choose to span symmetrically around their center. The length of the firm's production scope ($|\omega|$) is determined by problem (16),

$$i^*(\omega, m)X'(|\omega|)\mathbb{1}_k(m)[(1-(1-\theta)b)B(m)\gamma + b(1-\theta)r\delta a g^{\frac{\zeta}{\zeta+\lambda}} - a] = \mu|\omega|^{\iota}. \tag{50}$$

If $\mathbb{1}_k(m) = 0$, $i^*(\omega, m)$ and $D(\omega, m, z; \mathbf{z})$ do not depend on the firm's production scope. Due to the management cost, firms always choose $\omega = 0$.

Combining the two cases of $\mathbb{1}_k(m)$, the solution to (50) is only a function of m, i.e, $|\omega^*(m, z; \mathbf{z})| = \Omega(m)$ (Point 4).

Plugging in the solution of $i^*(\omega, m)$ and $\omega^*(m, z; \mathbf{z})$ into the government budget constraint derives (Point 6)

$$T = \sigma(\alpha_H C^i(i(\Omega(m_H), m_H)) + \alpha_L C^i(i(\Omega(m_L), m_L))). \tag{51}$$

The number of buyers of each type, (n_{bH}, n_{bL}), are the share of firms in each type that do not get an innovation output matching their production scope. The total number of buyers is the summation of the buyers of the two types. They are expressed as (Point 9)

$$n_{bH} = \alpha_H(1 - i^*(\omega^*(m_H), m_H)X(\omega^*(m_H))\mathbb{1}_k(m_H)); \tag{52}$$

$$n_{bL} = \alpha_H(1 - i^*(\omega^*(m_L), m_L)X(\omega^*(m_L))\mathbb{1}_k(m_L)); \tag{53}$$

$$n_b = n_{bH} + n_{bL}. \tag{54}$$

The number of agents is the summation of surviving patents from the previous period and the amount of firms that successfully innovate, but the output falls outside of their own production scope in the current period,

$$\begin{aligned} n_a = &\delta n_a(1-s) + \alpha_H i^*(\omega^*(m_H), m_H)(1 - X(\omega^*(m_H))\mathbb{1}_k(m_H)) \\ &+ \alpha_L i^*(\omega^*(m_L), m_L)(1 - X(\omega^*(m_L))\mathbb{1}_k(m_L)). \end{aligned} \tag{55}$$

Solving the equation above derives the expression for n_a in Point 9.

The value of the firm at the beginning of the period, $V(m, z; \mathbf{z})$, can be expressed as

$$V(m, z; \mathbf{z}) = D(\Omega(m), m, z; \mathbf{z}) - C^e(\omega; \mathbf{z}) \equiv v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}}, \tag{56}$$

where

$$v_1(m) = B(m); \tag{57}$$

$$v_2(m) = [\frac{(1-\sigma)\rho}{1+\rho}\chi i^*(\omega, m)^{1+\rho} + b(1-\theta)(B(m)\gamma - r\delta a g^{\frac{\zeta}{\zeta+\lambda}}) \tag{58}$$

$$+ r\mathbb{E}v_2(m')g^{\frac{\zeta}{\zeta+\lambda}} - \frac{\mu|\Omega(m)|^{1+\iota}}{1+\iota}]. \tag{59}$$

Since both $v_1(m)$ and $v_2(m)$ are only functions of m, the value function, $V(m, z; \mathbf{z})$, is consistent with the conjecture (Point 7).
The value of an agent is equal to the expected value of patent sales in the current period plus the continuation value, i.e.,

$$a\tilde{\mathbf{z}} = s[\frac{n_{bH}}{n_b}p_b(m_H, z; \mathbf{z}) + \frac{n_{bL}}{n_b}p_b(m_L, z; \mathbf{z})] + (1-s)r\delta a g^{\frac{\zeta}{\zeta+\lambda}}\tilde{\mathbf{z}} \tag{60}$$

Solving it derives the expression for a,

$$a = \frac{s[\frac{n_{bH}}{n_b}\theta B(m_H)\gamma + \frac{n_{bL}}{n_b}\theta B(m_L)\gamma]}{1-(1-s\theta)r\delta g^{\frac{\zeta}{\zeta+\lambda}}} \tag{61}$$

Since a is a constant, the value of an agent is linear in the aggregate innovation level (Point 7). The zero-profit condition of the agent requires that the price at which it collects patents, q, is equal to its value.

The representative household's problem can be expressed as

$$W(a; \mathbf{z}) = \max_{c, a'} u(c) + \beta W(a'; \mathbf{z})$$
$$s.t., c + a' = \frac{1}{r}a + \Pi,$$

where a is the asset holding of the household in the current period; $\frac{1}{r}$ is the capital return rate, where its relationship with the capital cost, $\tilde{r}$, is $\tilde{r} = \frac{1}{r} - 1 + \delta$; Π is the total profit of firms in this economy. Because all firms are owned by the household, the total profit is a part of the household's income. Solving the problem derives the following relationship on consumption across periods,

$$\frac{c'}{c} = (\frac{\beta}{r})^{\frac{1}{\epsilon}}. \tag{62}$$

Since consumption grows at the same rate, $g^{\frac{\zeta}{\zeta+\lambda}}$, as the total output, and the interest rate is fixed over time, we have (Point 2)

$$r = \frac{\beta}{g^{\epsilon\zeta/(\zeta+\lambda)}}. \tag{63}$$

The growth rate of the employment-weighted average innovation level of the economy, g, can be expressed by the following equation according to the definition,

$$g \equiv \frac{\alpha_H m_H \mathbf{z_H}' + \alpha_L m_L \mathbf{z_L}'}{\alpha_H m_H \mathbf{z_H} + \alpha_L m_L \mathbf{z_L}}. \tag{64}$$

In the balanced growth path equilibrium, the ratio of the innovation level of firms with high production ability to that of the firms with low production ability should be stable across periods, i.e.,

$$\frac{\mathbf{z_H}'}{\mathbf{z_L}'} = \frac{\mathbf{z_H}}{\mathbf{z_L}} \equiv o, \tag{65}$$

where o is a constant. Then (64) implies that

$$g = \frac{\mathbf{z_H}'}{\mathbf{z_H}} = \frac{\mathbf{z_L}'}{\mathbf{z_L}}. \tag{66}$$

Equations in (66) show that the growth rate in the innovation level of the aggregate economy is the same as the growth rate of firms across types.

The change in the average innovation levels of high- and low-type firms consists of two components.
(i) There is a reshuffling of firms at the beginning of each period because of the transition of production ability.
(ii) Firms update their innovation level through R&D or trade of patents.

The average innovation level of each type of firms after the transition of production ability but before the innovation stage in this period can be expressed as follows,

$$\mathbf{z_{Hr}} \equiv \frac{\alpha_H q_{HH} \mathbf{z_H} + \alpha_L q_{LH} \mathbf{z_L}}{\alpha_H q_{HH} + \alpha_L q_{LH}}; \tag{67}$$

$$\mathbf{z_{Lr}} \equiv \frac{\alpha_L q_{LL} \mathbf{z_L} + \alpha_H q_{HL} \mathbf{z_H}}{\alpha_L q_{LL} + \alpha_H q_{HL}}. \tag{68}$$

Firms update their innovation level in the R&D or trading process following the law of motion in (1). So, the growth rate of each type of firms in this process (denoted as g_H and g_L) depends on the share of them that use their own innovation output and the share that successfully buy a patent on the market.

$$\begin{aligned} g_H \equiv \frac{\mathbf{z_H}'}{\mathbf{z_{Hr}}} = 1 + [&i^*(\omega^*(m_H), m_H) X(\omega^*(m_H)) \mathbb{1}_k(m_H) \\ &+ (1 - i^*(\omega^*(m_H), m_H) X(\omega^*(m_H)) \mathbb{1}_k(m_H) b)] \gamma \frac{\mathbf{z}}{\mathbf{z_{Hr}}}; \end{aligned} \tag{69}$$

$$\begin{aligned} g_L \equiv \frac{\mathbf{z_L}'}{\mathbf{z_{Lr}}} = 1 + [&i^*(\omega^*(m_L), m_L) X(\omega^*(m_L)) \mathbb{1}_k(m_L) \\ &+ (1 - i^*(\omega^*(m_L), m_L) X(\omega^*(m_L)) \mathbb{1}_k(m_L)) b] \gamma \frac{\mathbf{z}}{\mathbf{z_{Lr}}}. \end{aligned} \tag{70}$$

Using the relationship $\mathbf{z_H}' = g_H \mathbf{z_{Hr}}$ and plugging equations (65), (67), (68), (69) and (70) into the first equation in (66) derive the solutions for g and o through the following system of equations,

$$g = \frac{g_H(\alpha_H q_{HH} + \alpha_L q_{LH} \frac{1}{o})}{\alpha_H q_{HH} + \alpha_L q_{LH}}; \tag{71}$$

$$o = \frac{g_H(\alpha_H q_{HH} o + \alpha_L q_{LH})}{\alpha_H q_{HH} + \alpha_L q_{LH}} \frac{\alpha_L q_{LL} + \alpha_H q_{HL}}{g_L(\alpha_L q_{LL} + \alpha_H q_{HL} o)}. \tag{72}$$

Since all of the other variables and parameters are fixed in the equation system, the solutions of g and o are indeed both constants (Point 1). □

C.2 Characterization of Proposition 7.1

There exists a symmetric balanced growth path of the form:
1. The employment-weighted growth rate of the aggregate innovation level, g, and the ratio of the average innovation level of firms with high production ability to that of firms with low production

ability, o, defined by,

$$g = \frac{\int\int m'z''dF(m',z')/\int\int m'dF(m',z')}{\int\int mz'dF(m,z)/\int\int mdF(m,z)};\; o = \frac{\int z'dF(m,z)|_{m=m_H}}{\int z'dF(m,z)|_{m=m_L}},$$

are constants.
2. *The interest factor* $r = \beta/g^{\epsilon\zeta/(\zeta+\lambda)}$; *the rental rate on capital* $\tilde{r} = g^{\epsilon\zeta/(\zeta+\lambda)}/\beta - 1 + \delta$.
3. *The odds of a successful match for a potential buyer,* $b^j(\omega)$, *and for a potential seller,* s^j, *on the market of each type (basic or applied) of patents, only depend on the total number of patent buyers and sellers on that market, i.e.,* $b^j(\omega) = \phi(\frac{n_a^j}{n_b^j})^\nu$, $s^j = \phi(\frac{n_b^j}{n_a^j})^{1-\nu}$, *where* $j \in \{a, b\}$.
4. *The production scope of each firm spans symmetrically around the center, and the length of the scope depends only on the production ability of the firm, i.e.,* $|\omega(m, z; \mathbf{z})| = \Omega(m)$.
5. *The success rates of applied and basic research do not depend on the firm's innovation level,* z, *or the economy-wide innovation level,* $\mathbf{z}$, *i.e.,* $i^j(\omega, m, z; \mathbf{z}) = i^j(\omega, m),\ j \in \{a, b\}$.
6. *The government budget constraint is,*

$$T = \sigma \sum_{j\in\{a,b\}} (\alpha_H C^{ij}(i^j(\Omega(m_H), m_H)) + \alpha_L C^{ij}(i^j(\Omega(m_L), m_L))).$$

7. *The value function* $V(m, z; \mathbf{z})$ *is linear in* $\tilde{z}$ *and* $\tilde{\mathbf{z}}$, *i.e.,* $V(m, z; \mathbf{z}) = v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}}$. *The value of a type-j agent is linear in* $\tilde{\mathbf{z}}$, *i.e.,* $A^j(\mathbf{z}) = a^j\tilde{\mathbf{z}}$. $\tilde{z} = z/\mathbf{z}^{\lambda/(\zeta+\lambda)}$ *and* $\tilde{\mathbf{z}} = \mathbf{z}^{\zeta/(\zeta+\lambda)}$.
8. *Keeping or selling a within-scope innovation only depends on the firm's production ability, m.*
9. *The number of buyers of both types* (n_{bH}^j, n_{bL}^j) *and the number of sellers* (n_a^j) *for j* $(j \in \{a, b\})$ *type of patents are*

$$n_{bH}^j = \alpha_H(1 - i^{j*}(\omega^*(m_H), m_H)X^j(\omega^*(m_H))\mathbb{1}_k^j(m_H));$$
$$n_{bL}^j = \alpha_H(1 - i^{j*}(\omega^*(m_L), m_L)X^j(\omega^*(m_L))\mathbb{1}_k^j(m_L));$$
$$n_a^j = \frac{\alpha_H i^{j*}(\omega^*(m_H), m_H)(1 - X^j(\omega^*(m_H))\mathbb{1}_k^j(m_H)) + \alpha_L i^{j*}(\omega^*(m_L), m_L)(1 - X^j(\omega^*(m_L))\mathbb{1}_k^j(m_L))}{1 - \delta(1-s)}.$$

10. *The buying price and the expected selling price of a j-type* $(j \in \{a, b\})$ *patent is*

$$p_b^j(m, z; \mathbf{z}) = \theta(Jm + \frac{r}{g^{\lambda/(\lambda+\zeta)}}\mathbb{E}[v_1(m')|m])\gamma^j\tilde{\mathbf{z}} + (1-\theta)r\delta a^j g^{\frac{\zeta}{\zeta+\lambda}}\tilde{\mathbf{z}};$$
$$q^j(\mathbf{z}) = A^j(\mathbf{z}) = a^j\tilde{\mathbf{z}},$$

where J is a constant.

C.3 Proof of Proposition 7.1

Proof. The proof is very similar to that of Proposition 5.1. One difference is that the profit of each type of firms now have four possible cases. (i) The firm gets both applied and basic R&D output (either through own innovation or purchasing them from the market). The profit in this case is $\pi(m, z^{ab}; \mathbf{z}) = Jm(\tilde{z} + \gamma^a \tilde{\mathbf{z}} + \gamma^b \tilde{\mathbf{z}})$. (ii) The firm gets only applied R&D output. The profit is $\pi(m, z^a; \mathbf{z}) = Jm(\tilde{z} + \gamma^a \tilde{\mathbf{z}})$. (iii) The firm gets only basic R&D output. The profit is $\pi(m, z^b; \mathbf{z}) = Jm(\tilde{z} + \gamma^b \tilde{\mathbf{z}})$. (iv). The firm gets neither R&D output. The profit is $\pi(m, z; \mathbf{z}) = Jm(\tilde{z})$. $J = \zeta(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta+\lambda}}[(\alpha_H m_H + \alpha_L m_L)g]^{-\frac{\lambda}{\zeta+\lambda}}$ for all the four cases.

Then, from the Nash bargaining problem between the buyer and the seller, it can be derived that for a j-type patent ($j \in \{a, b\}$), the buying price can be expressed as

$$p_b^j(m; \mathbf{z}) = \theta B(m)\gamma^j \tilde{\mathbf{z}} + (1-\theta)r\delta a^j g^{\frac{\zeta}{\zeta+\lambda}} \tilde{\mathbf{z}}, \tag{73}$$

where $B(m) = [Jm + r\mathbb{E}(v_1(m'))g^{-\frac{\lambda}{\zeta+\lambda}}]$.
The value of an agent that buys a type-j innovation is

$$A^j(\mathbf{z}) = a^j \tilde{\mathbf{z}} = \frac{s^j[\frac{n^j_{bH}}{n^j_b}\theta B(m_H)\gamma^j + \frac{n^j_{bL}}{n^j_b}\theta B(m_L)\gamma^j]}{1 - (1 - s\theta)r\delta g^{\frac{\zeta}{\zeta+\lambda}}} \tilde{\mathbf{z}} \tag{74}$$

The zero-profit condition of the agent requires that the price at which it collects patents, $q^j(\mathbf{z})$, is equal to $A^j(\mathbf{z})$.

The optimal success rate of the j-type R&D ($j \in \{a, b\}$) is

$$\begin{aligned} i^{j*}(\omega, m) =& \{\frac{1}{(1-\sigma)\chi^j}[X^j(\omega)\mathbb{1}^j_k(m)((1-(1-\theta)b^j)B(m)\gamma^j + b^j(1-\theta)r\delta a^j g^{\frac{\zeta}{\zeta+\lambda}}) \\ & + (1 - X^j(\omega)\mathbb{1}^j_k(m))a^j]\}^{\frac{1}{\rho}}, \end{aligned} \tag{75}$$

which also only depends on the firm's production scope and production ability.

If the firm keeps at least one within-scope innovation, i.e., $\sum_{j\in\{a,b\}} \mathbb{1}^j_k(m) \geq 1$, the length of the firm's production scope is determined by,

$$\sum_{j\in\{a,b\}} i^{j*}(\omega, m)X^{j'}(|\omega|)\mathbb{1}^j_k(m)[(1-(1-\theta)b^j)B(m)\gamma^j + b^j(1-\theta)r\delta a^j g^{\frac{\zeta}{\zeta+\lambda}} - a^j] = \mu|\omega|^\iota. \tag{76}$$

Otherwise, the firm chooses $\omega = 0$. In both cases, the optimal scope is only a function of m.

The growth rates of each type of firms in the R&D and search and matching stages are respectively

$$g_H \equiv \frac{\mathbf{z_H}'}{\mathbf{z_{Hr}}} = 1 + \sum_{j\in\{a,b\}} [i^{j*}(\omega^*(m_H), m_H)X^j(\omega^*(m_H))\mathbb{1}_k^j(m_H) \\ + (1 - i^{j*}(\omega^*(m_H), m_H)X^j(\omega^*(m_H))\mathbb{1}_k^j(m_H))b^j]\gamma^j \frac{\mathbf{z}}{\mathbf{z_{Hr}}}; \tag{77}$$

$$g_L \equiv \frac{\mathbf{z_L}'}{\mathbf{z_{Lr}}} = 1 + \sum_{j\in\{a,b\}} [i^{j*}(\omega^*(m_L), m_L)X^j(\omega^*(m_L))\mathbb{1}_k^j(m_L) \\ + (1 - i^{j*}(\omega^*(m_L), m_L)X^j(\omega^*(m_L))\mathbb{1}_k^j(m_L))b^j]\gamma^j \frac{\mathbf{z}}{\mathbf{z_{Lr}}}. \tag{78}$$

Still, the growth rate in the social innovation level and the ratio of the innovation levels between high- and low-type firms are constants and equal to

$$g = \frac{g_H(\alpha_H q_{HH} + \alpha_L q_{LH} \frac{1}{o})}{\alpha_H q_{HH} + \alpha_L q_{LH}}; \tag{79}$$

$$o = \frac{g_H(\alpha_H q_{HH} o + \alpha_L q_{LH})}{\alpha_H q_{HH} + \alpha_L q_{LH}} \frac{\alpha_L q_{LL} + \alpha_H q_{HL}}{g_L(\alpha_L q_{LL} + \alpha_H q_{HL} o)}. \tag{80}$$

□

D Calibration

D.1 Estimation of the Matching Elasticity

Table 15 displays the estimation results of the elasticity in the matching function of the patent trading market. The first three columns use raw numbers, while the last three columns use patent citation-weighted numbers. The numbers are summed at the 6-digit NAICS code level in columns (1) and (4); at the 4-digit NAICS code level in columns (2) and (5); at the 2-digit NAICS code level in columns (3) and (6). In most columns, the summation of the two coefficients is not far from 1, suggesting that the matching function is close to being constant-return-to-scale. The coefficient of the number of sellers, which corresponds to the matching elasticity(ν), is in the range of 0.598-0.821. The calibration then sets the value of ν as 0.70.

Table 15: Estimation of the Elasticity in the Matching Function

	Ln(Number of Matches)					
	(1)	(2)	(3)	(4)	(5)	(6)
		Raw			Citation-Weighted	
Ln(Num. of Sellers)	0.598	0.693	0.780	0.604	0.694	0.821
	(0.006)	(0.012)	(0.049)	(0.006)	(0.012)	(0.050)
Ln(Num. of Buyers)	0.0713	0.105	0.291	0.0698	0.102	0.222
	(0.008)	(0.018)	(0.089)	(0.008)	(0.018)	(0.090)
Observations	20000	5700	500	20000	5700	500
R-squared	0.873	0.936	0.984	0.871	0.935	0.983

Notes: The dependent variable is the logarithm of the number of matches at different level of sectors. The numbers are at the 6-digit NAICS code level in columns (1) and (4); at the 4-digit NAICS code level in columns (2) and (5); at the 2-digit NAICS code level in columns (3) and (6). Columns (1)-(3) use raw numbers, while columns(4)-(6) use patent citation-weighted numbers. The number of observations is rounded to the nearest 100 to comply with the disclosure requirement of the Census Bureau.

Table 16: Relationship of the Within-Scope Probability and the Number of Industries

VARIABLE	Log(Within-Scope Probability)
Ln(Num. of Industries)	0.7643
	(0.0134)
Constant	-4.443
	(0.0370)
Observations	150
R-squared	0.9547

Notes: Firms are grouped by the number of 6-digit NAICS codes they have. The dependent variable is the average likelihood that firms' patents match their production in each group. The independent variable is the logarithm of the number of 6-digit NAICS codes in each group. The number of observations is rounded to the nearest 50 to comply with the disclosure requirement of the Census Bureau.

D.2 Estimation of the Within-scope Probability Function

Table 16 shows the estimation of the within-scope probability function ($X(\omega)$). To avoid disclosure of the information of specific firms, firms are grouped by the number of 6-digit NAICS codes they have. Then the average likelihood that firms' patents match their production is calculated for each group. Then, $X(\omega)$ is estimated by running regressions of the likelihood on the number of industries. The high R-squared confirms that the function form assumed in the model can capture the actual relationship.

D.3 Decomposition of the Growth Rate

According to Equation (71), the social average growth rate in the innovation level depends on the growth rate of firms with high production ability, g_H, and the ratio of innovation levels between firms with high and low production ability, o. Further, Equation (69) suggests that g_H depends on the lock-step updating rule of the innovation level from R&D and patent trade, and the term, $\frac{\mathbf{z}}{\mathbf{z_{Hr}}}$. Since the term, $\frac{\mathbf{z}}{\mathbf{z_{Hr}}}$, only depends on o and exogenous parameters, the social average growth rate, g, can be separated into two parts—the innovation level increase from R&D and patent trade, and the reallocation of resources across firm sizes that results in a change in the relative innovation level, o. The contribution of the former source can be obtained by fixing o while changing the R&D and patent trade process; the contribution of the latter source can be obtained by fixing the R&D and patent trade process while changing o.

D.4 Calibration of the Extended Model

This paper calibrates the newly added parameters, $\{\chi^j, \rho^j, \gamma^j\}$, and the two probability functions, $X^j(.)$, where j is an indicator of basic or applied research, in the following way. The ratio of the step sizes, $\frac{\gamma_b}{\gamma_a}$, is set to be consistent with Akcigit, Hanley and Serrano-Velarde (2021). The within-scope probability functions are estimated by the same method as the estimation of $X(.)$ in the baseline model, except that the regression is run on two separate samples—patents from basic research and patents from applied research or development in the SIRD. The scale parameter of the applied research cost function (χ^a) is normalized to be 1. The scale parameter of the basic research cost function (χ^b), the step size of applied research (γ_a), and the two elasticities (ρ^a, ρ^b) are pinned down together with $\{\phi, \theta, \mu, \iota\}$ in the calibration. Two additional moments are added—the share of basic research expense in total R&D expense, respectively, for firms with high and low production ability. All the other parameters are disciplined by the method used to calibrate the baseline model, and the decomposition method is the same as before. Table 17 shows the results. The estimated within-scope probability functions suggest that when the industry number of a firm is not too large, it is harder for basic research output to match the firm's production compared to applied research. In the calibration, the annual growth rate is mostly affected by γ_a; the basic research share and the R&D cost to domestic sales ratio of firms with high and low production ability are mostly governed by χ_b, $1+\rho_a$, and $1+\rho_b$.

The extended model is calibrated to both the initial and the ending balanced growth paths. In this process, parameters corresponding to the four mechanisms, $\{\phi, \theta, \sigma, \mu, \iota, \gamma_a, \chi_b, \rho_a, \rho_b\}$, are changed to match the data moments in the two periods.

The model fit of the two balanced growth paths are shown respectively in Table 18 and Table 19. Overall, the model matches the data well.

Table 17: Parameter Values of the Extended Model

Parameter	Description	Value	Identification
Priori Info.			
$\frac{\gamma_b}{\gamma_a}$	Step Size Ratio	1.6	Akcigit et al. (2021)
χ_a	Applied R Cost, Scale	1	Normalization
Estimation			
$X^a(\omega)$	Applied R, Within-Scope Prob.	$e^{-3.837} * \|\omega\|^{0.602}$	Regression
$X^b(\omega)$	Basic R, Within-Scope Prob.	$e^{-4.944} * \|\omega\|^{0.932}$	Regression
Model			
γ_a	Applied R Step Size	1.08	Growth Rate
χ_b	Basic R Cost, Scale	4.15	Basic Research Share,
$1+\rho_a$	Applied R Cost, Elasticity	1.66	R&D Cost/Sales
$1+\rho_b$	Basic R Cost, Elasticity	1.19	Ratio (H and L)

Notes: The newly added parameters are calibrated by a priori information, direct estimation, and minimizing the distance between the model and data moments. When calculating the minimized distance, the new parameters are jointly calibrated with the old parameters in Table 4.

Table 18: Model Fit for Key Moments in the Initial Balanced Growth Path

Targets	Data	Model
Economic Growth Rate(1981-1985)	2.13%	2.13%
R&D Cost/Sales of H Firms(1981-1985)	3.62%	3.62%
R&D Cost/Sales of L Firms(1981-1985)	2.83%	2.83%
Basic R Share of H Firms(1981-1985)	4.20%	4.20%
Basic R Share of L Firms(1981-1985)	3.73%	3.73%
Avg. Number of Industries of H Firms(1981-1985)	11.81	11.81
Avg. Number of Industries of L Firms(1981-1985)	1.92	1.92
The Share of Patents Transacted(1983)	23.2%	23.2%

Notes: The model and data moments in the initial balanced growth path are almost the same, showing the model fits the data well.

E Supplementary Materials for Empirical Analysis

E.1 Summary Statistics

Panel A and B in Table 20 respectively show summary statistics of the regression samples for production scope and R&D intensity.[45] The number of industries per firm experiences a decrease after the CAFC (Post=1), while the average employment remains at nearly the same level. The average share of employment in the two highly treated industries is around 2%. The overall R&D intensity increases after the CAFC (Post=1). The common control variables are comparable in magnitude in the two panels. The average invalida-

[45] The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

Table 19: Model Fit for Key Moments in the Ending Balanced Growth Path

Targets	Data	Model
Economic Growth Rate(1996-2000)	2.22%	2.22%
R&D Cost/Sales of H Firms(1996-2000)	3.15%	3.15%
R&D Cost/Sales of L Firms(1996-2000)	6.71%	6.71%
Basic R Share of H Firms(1996-2000)	4.61%	4.61%
Basic R Share of L Firms(1996-2000)	11.46%	11.46%
Avg. Number of Industries of H Firms(1996-2000)	6.31	6.31
Avg. Number of Industries of L Firms(1996-2000)	1.61	1.61
The Share of Patents Transacted(2000)	37.0%	37.0%

Notes: The model and data moments in the ending balanced growth path are almost the same, showing the model fits the data well.

tion rate across different regions is around 54%.[46] There is a drop in the federal corporate income tax rate and a rise in both the federal and state-level R&D tax credits.

E.2 Placebo Tests

It is possible that the differential changes in the number of industries and R&D intensity across regions and firms are due to pre-trends instead of the policy impact. To check whether there are pre-existing trends, this study runs the same regressions in Equation (22)–(24) on the pre-CAFC sample (1976-1982). All variables are defined as the same as before, except the *post* dummy. Now, the *post* dummy (written as *post*2) equals zero if the observation year is before or in 1979; equals one if after 1979.[47] If there are pre-trends in production scope, β in Equation (22) and β_1 and β_2 in Equation (23) should still be significantly negative. However, as shown in Table 21 and Table 22, they are either positive or tiny in absolute magnitude. None of them is significant, showing that the differential changes in production scope are not due to pre-existing trends.

If there are pre-trends in the R&D intensity, β_1 in Equation (24) should be still positive while β_2 still negative. However, as shown in Table 23, their signs are flipped, showing that the differential changes in R&D intensity are not due to pre-existing trends. Therefore, the empirical results in section 8.4 can be viewed as evidence of causality from the policy reforms to firms' shrinkage in production scope and reallocation of R&D activities.

[46] There is very little change in this rate before and after the CAFC because both of them are at the pre-CAFC level.

[47] This study also tries other ways of segmenting the pre-CAFC sample. The results are similar.

Table 20: Summary Statistics of the Regression Sample

Sample	Mean			Standard Deviation		
	All	Post=0	Post=1	All	Post=0	Post=1
Panel A						
Observations	268000	131000	136000	268000	131000	136000
Number of Industries	3.066	3.074	3.058	6.722	6.952	6.494
Employment	1187	1187	1187	9670	10780	8467
Highly Treated Share	0.02101	0.01987	0.0221	0.1337	0.129	0.138
Pre-CAFC Invalid. Rate	0.5375	0.5381	0.5369	0.1082	0.1082	0.1083
Real GDP	144000	127200	160200	115000	95460	129100
Effective Federal Tax Rate	0.4105	0.4335	0.3883	0.0434	0.01645	0.04934
Effective State Tax Rate	0.07406	0.07325	0.07484	0.02676	0.0279	0.02558
Federal R&D Tax Credits	0.01443	0.004603	0.02388	0.01145	0.007372	0.004747
State R&D Tax Credits	0.0006073	0.0001753	0.001023	0.003604	0.002553	0.004343
Panel B						
Observations	41000	20000	21000	41000	20000	21000
Sum of Weight	220000	100000	120000	220000	100000	120000
R&D Intensity	0.1268	0.06814	0.176	0.9789	0.4915	1.247
Employment	1355	1094	1574	13570	9913	16000
Small Firm Share	0.8989	0.8956	0.9017	0.3014	0.3058	0.2977
Pre-CAFC Invalid. Rate	0.5387	0.5446	0.5338	0.1103	0.1089	0.1113
Real GDP	146500	129300	161000	121600	99300	135900
Effective Federal Tax Rate	0.4068	0.4339	0.3839	0.04653	0.01692	0.05101
Effective State Tax Rate	0.07348	0.07321	0.0737	0.02763	0.02926	0.02617
Federal R&D Tax Credits	0.01473	0.004456	0.02336	0.01114	0.007208	0.004627
State R&D Tax Credits	0.0006286	0.0001987	0.0009896	0.002836	0.002718	0.002883

Notes: Panel A shows the summary statistics of the regression sample for production scope. The sample contains the innovating firms in the LBD that existed before or in 1982, the year of the establishment of the CAFC. Panel B shows the summary statistics of the regression sample for R&D intensity. The sample contains all the firms in the SIRD that existed before or in 1982. the R&D intensity regression is weighted by the sample weight assigned to each observation in the SIRD. The sample period for all regressions is from 1976 to 1989, 7 years before and after the reform. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

Table 21: Placebo Test-DiD Regression on Production Scope

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
Invalidation Rate*Post2	0.00196	0.0206	0.000678	-0.00194
	(0.013)	(0.014)	(0.013)	(0.012)
Ln(Employment)	0.0539***	0.0529***	0.0526***	0.0527***
	(0.003)	(0.003)	(0.003)	(0.003)
Real GDP	NO	YES	NO	YES
Tax Rates	NO	YES	NO	YES
R&D Tax Credits	NO	YES	NO	YES
Post Dummy	YES	YES	NO	NO
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	131000	131000	131000	131000
R-squared	0.97	0.97	0.97	0.97

Notes: The dependent variable is the logarithm of the number of 6-digit NAICS codes owned by the firm. The four columns have different control variables. Standard errors are clustered by circuit court regions $\times$ the post dummy. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

Table 22: Placebo Test-DDD Regression on Production Scope

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
High_treat*Invalidation Rate*Post2	0.00965	0.00894	0.011	0.0121
	(0.038)	(0.038)	(0.038)	(0.039)
Invalidation Rate*Post2	0.00177	0.0204	0.000479	-0.00213
	(0.012)	(0.014)	(0.012)	(0.012)
High_treat*Post2	-0.0094	-0.00877	-0.00935	-0.00908
	(0.020)	(0.020)	(0.020)	(0.020)
Ln(Employment)	0.0539***	0.0530***	0.0526***	0.0527***
	(0.003)	(0.003)	(0.003)	(0.003)
Real GDP	NO	YES	NO	YES
Tax Rates	NO	YES	NO	YES
R&D Tax Credits	NO	YES	NO	YES
Post Dummy	YES	YES	NO	NO
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	131000	131000	131000	131000
R-squared	0.97	0.97	0.97	0.97

Notes: The dependent variable is the logarithm of the number of 6-digit NAICS codes owned by the firm. The four columns have different control variables. Standard errors are clustered by circuit court regions $\times$ the post dummy. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

Table 23: Placebo Test-DDD Regression on R&D Intensity

Dependent Variable	R&D Expenses to Domestic Sales Ratio			
	(1)	(2)	(3)	(4)
Small*Invalidation Rate*Post2	-0.0467	-0.057	-0.0417	-0.057
	(0.048)	(0.052)	(0.048)	(0.052)
Invalidation Rate*Post2	0.0772*	0.0994**	0.0734*	0.0994**
	(0.039)	(0.049)	(0.039)	(0.049)
Small*Post2	0.042	0.0584*	0.0497*	0.0584*
	(0.028)	(0.031)	(0.029)	(0.031)
Ln(Employment)	0.00197	0.0045	0.00435	0.0045
	(0.024)	(0.025)	(0.025)	(0.025)
Real GDP	NO	YES	NO	YES
Tax Rates	NO	YES	NO	YES
R&D Tax Credits	NO	YES	NO	YES
Post Dummy	YES	YES	NO	NO
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations (Weighted)	100000	100000	100000	100000
R-squared	0.853	0.853	0.853	0.853

Notes: The dependent variable is the firm's R&D-expenses-to-domestic-sales ratio. The four columns have different control variables. Standard errors are clustered by circuit court regions $\times$ the post dummy. The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.